Name:

Venn Diagrams
Showing Probabilities
Date:

Time:

## Total marks available:

Total marks achieved: $\qquad$

## Questions

Q1.
(a) State in words the relationship between two events $R$ and $S$ when $P(R \cap S)=0$

The events $A$ and $B$ are independent with $P(A)=\frac{1}{4}$ and $P(A \cup B)=\frac{2}{3}$

Find
(b) $\mathrm{P}(B)$
(c) $\mathrm{P}\left(A^{\prime} \cap B\right)$
(d) $P\left(B^{\prime} \mid A\right)$

Q2.
Jake and Kamil are sometimes late for school.
The events $J$ and $K$ are defined as follows
$J=$ the event that Jake is late for school
$K=$ the event that Jake is late for school
$\mathrm{P}(J)=0.25, \mathrm{P}(J \cap K)=0.15$ and $\mathrm{P}\left(J ' \cap K^{\prime}\right)=0.7$
On a randomly selected day, find the probability that
(a) at least one of Jake or Kamil are late for school,
(b) Kamil is late for school.

Given that Jake is late for school,
(c) find the probability that Kamil is late.

The teacher suspects that Jake being late for school and Kamil being late for school are linked in some way.
(d) Determine whether or not $J$ and $K$ are statistically independent.
(e) Comment on the teacher's suspicion in the light of your calculation in (d).

Q3.
(a) Given that $\mathrm{P}(A)=a$ and $\mathrm{P}(B)=b$ express $\mathrm{P}(A \cup B)$ in terms of $a$ and $b$ when
(i) $A$ and $B$ are mutually exclusive,
(ii) $A$ and $B$ are independent.

Two events $R$ and $Q$ are such that
$\mathrm{P}\left(R \cap Q^{\prime}\right)=0.15, \quad \mathrm{P}(Q)=0.35$ and $\mathrm{P}(R \mid Q)=0.1$
Find the value of
(b) $\mathrm{P}(R \cup Q)$,
(c) $\mathrm{P}(R \cap Q)$,
(d) $\mathrm{P}(R)$.

Q4.
Given that

$$
\mathrm{P}(A)=0.35, \mathrm{P}(B)=0.45 \text { and } \mathrm{P}(A \cap B)=0.13
$$

find $(A \cup B)$
(b) $P\left(A^{\prime} \mid B^{\prime}\right)$

The event $C$ has $\mathrm{P}(C)=0.20$
The events $A$ and $C$ are mutually exclusive and the events $B$ and $C$ are independent.
(c) Find $\mathrm{P}(B \cap C)$
(d) Draw a Venn diagram to illustrate the events $A, B$ and $C$ and the probabilities for each region.

Tom invites Avisha to play a game with these dice.
(e) Find $\mathrm{P}\left([B \cup C]^{\prime}\right)$
(Total 12 marks)

Q5.


Figure 1
The Venn diagram in Figure 1 shows three events $A, B$ and $C$ and the probabilities associated with each region of $B$. The constants $p, q$ and $r$ each represent probabilities associated with the three separate regions outside $B$.

The events $A$ and $B$ are independent.
(a) Find the value of $p$.

Given that $\mathrm{P}(B \mid C)=\frac{5}{11}$
(b) find the value of $q$ and the value of $r$.
(c) Find $P(A \cup C \mid B)$.

Q6.
For the events $A$ and $B$,

$$
P\left(A^{\prime} \cap B\right)=0.22 \text { and } P\left(A^{\prime} \cap B^{\prime}\right)=0.18
$$

(a) Find $P(A)$.
(b) Find $\mathrm{P}(A \cup B)$.

Given that $P(A \mid B)=0.6$
(c) find $\mathrm{P}(A \cap B)$.
(d) Determine whether or not $A$ and $B$ are independent.

Q7.
$A$ and $B$ are two events such that

$$
P(B)=1 / 2 \quad P(A \mid B)=2 / 5 \quad P(A \cup B)=13 / 20
$$

(a) Find $\mathrm{P}(A \cap B)$.
(b) Draw a Venn diagram to show the events $A, B$ and all the associated probabilities.

Find
(c) $P(A)$
(d) $\mathrm{P}(B \mid A)$
(e) $P\left(A^{\prime} \cap B\right)$

Q8.
The Venn diagram shows the probabilities of customer bookings at Harry's hotel.
$R$ is the event that a customer books a room
$B$ is the event that a customer books breakfast
$D$ is the event that a customer books dinner $u$ and $t$ are probabilities.

(a) Write down the probability that a customer books breakfast but does not book a room.

Given that the events $B$ and $D$ are independent
(b) find the value of $t$
(c) hence find the value of $u$
(d) Find
(i) $\mathrm{P}(D \mid R \cap B)$
(ii) $\mathrm{P}\left(D \mid R \cap B^{\prime}\right)$

A coach load of 77 customers arrive at Harry's hotel.
Of these 77 customers
40 have booked a room and breakfast
37 have booked a room without breakfast
(e) Estimate how many of these 77 customers will book dinner.

