Name: ____

Trigonometry Problems 2

Date:

Time:

Total marks available:

Total marks achieved: ______

Mark Scheme

Q1.

Question Number	Scheme	Marks
(a)	$N(2, -1)$	B1, B1 (2)
(b)	$r = \sqrt{\frac{169}{4}} = \frac{13}{2} = 6.5$	B1 (1)
(c)	Complete Method to find x coordinates, $x_2 - x_1 = 12$ and $\frac{x_1 + x_2}{2} = 2$ then solve	M1
	To obtain $x_1 = -4$, $x_2 = 8$	A1ft A1ft
	Complete Method to find y coordinates, using equation of circle or Pythagoras i.e. let d be the distance below N of A *then* $d^2 = 6.5^2 - 6^2 \Rightarrow d = 2.5 \Rightarrow y = ..$	M1
	So $y_2 = y_1 = -3.5$	A1 (5)
(d)	Let $A\hat{N}B = 2\theta \Rightarrow \sin\theta = \frac{6}{"6.5"} \Rightarrow \theta = (67.38)...$	M1
	So angle ANB is 134.8 *	A1 (2)
(e)	AP is perpendicular to AN so using triangle ANP $\tan\theta = \frac{AP}{"6.5"}$	M1
	Therefore $AP = 15.6$	A1cao (2)
		[12]
(a) (b)	B1 for 2 (α) , B1 for −1 B1 for 6.5 o.e.	
(c)	1st M1 for finding x coordinates – may be awarded if either x co-ord is correct A1ft,A1ft are for $\alpha - 6$ and $\alpha + 6$ if x coordinate of N is α 2nd M1 for a method to find y coordinates – may be given if y co-ordinate is correct A marks is for −3.5 only.	
(d)	M1 for a full method to find θ or angle ANB (eg sine rule or cosine rule directly or finding another angle and using angles of triangle.) **ft their 6.5 from radius or wrong *y*.** (cos ANB = $\frac{"6.5"^2 + "6.5"^2 - 12^2}{2 \times "6.5" \times "6.5"}$ =-0.704) A1 is a printed answer and must be 134.8 – do not accept 134.76.	
(e)	M1 for a full method to find AP Alternative Methods N.B. May use triangle AXP where X is the mid point of AB. Or may use triangle ABP. From circle theorems may use angle BAP = 67.38 or some variation. Eg $\frac{AP}{\sin 67.4} = \frac{12}{\sin 45.2}$, $AP = \frac{6}{\sin 22.6}$ or $AP = \frac{6}{\cos 67.4}$ are each worth M1	

Q2.

Question Number	Scheme	Marks
(a)	$11^2 = 8^2 + 7^2 - (2\times 8\times 7\cos C)$	M1
	$\cos C = \dfrac{8^2+7^2-11^2}{2\times 8\times 7}$ (or equivalent)	A1
	$\{\hat{C} = 1.64228...\} \Rightarrow \hat{C} = \text{awrt } 1.64$	A1 cso
		(3)
(b)	Use of Area $\Delta ABC = \frac{1}{2}ab\sin(\text{their } C)$, where a, b are any of 7, 8 or 11.	M1
	$= \frac{1}{2}(7\times 8)\sin C$ using the value of their C from part (a).	A1 ft
	$\{= 27.92848...$ or $27.93297...\}$ = awrt 27.9 (from angle of either 1.64^c or 94.1°)	A1 cso
		(3)
		[6]
	Notes	
(a)	M1 is also scored for $8^2 = 7^2 + 11^2 - (2\times 7\times 11\cos C)$ or $7^2 = 8^2 + 11^2 - (2\times 8\times 11\cos C)$ or $\cos C = \dfrac{7^2+11^2-8^2}{2\times 7\times 11}$ or $\cos C = \dfrac{8^2+11^2-7^2}{2\times 8\times 11}$ 1st A1: Rearranged correctly to make $\cos C = ...$ and numerically correct (possibly unsimplified). Award A1 for any of $\cos C = \dfrac{8^2+7^2-11^2}{2\times 8\times 7}$ or $\cos C = \dfrac{-8}{112}$ or $\cos C = -\dfrac{1}{14}$ or $\cos C = \text{awrt } -0.071$. SC: Also allow 1st A1 for $112\cos C = -8$ or equivalent. Also note that the 1st A1 can be implied for $\hat{C} = \text{awrt } 1.64$ or $\hat{C} = \text{awrt } 94.1^\circ$. **Special Case:** $\cos C = \dfrac{1}{14}$ or $\cos C = \dfrac{11^2-8^2-7^2}{2\times 8\times 7}$ scores a SC: M1A0A0. 2nd A1: for awrt 1.64 **cao** **Note that** $A = 0.6876...^c$ $(\text{or } 39.401...^\circ)$, $B = 0.8116...^c$ $(\text{or } 46.503...^\circ)$	
(b)	M1: alternative methods must be fully correct to score the M1. For any (or both) of the M1 or the 1st A1; their C can either be in degrees or radians. Candidates who use $\cos C = \dfrac{1}{14}$ to give $C = 1.499...$, can achieve the correct answer of awrt 27.9 in part (b). These candidates will score M1A1A0cso, in part (b). Finding $C = 1.499...$ in part (a) and achieving awrt 27.9 with no working scores M1A1A0. Otherwise with no working in part (b), awrt 27.9 scores M1A1A1. **Special Case: If the candidate gives awrt 27.9 from any of the below then award M1A1A1.** $\frac{1}{2}(7\times 11)\sin(0.8116^c \text{ or } 46.503^\circ) = \text{awrt } 27.9$, $\frac{1}{2}(8\times 11)\sin(0.6876...^c \text{ or } 39.401...^\circ) = \text{awrt } 27.9$. **<u>Alternative: Hero's Formula:</u>** $A = \sqrt{13(13-11)(13-8)(13-7)} = \text{awrt } 27.9$, where M1 is attempt to apply $A = \sqrt{s(s-11)(s-8)(s-7)}$ and the first A1 is for the correct application of the formula.	

Q3.

Question number	Scheme	Marks
	(a) $4^2 = 5^2 + 6^2 - (2\times5\times6\cos\theta)$	M1
	$\cos\theta = \frac{5^2+6^2-4^2}{2\times5\times6}$	A1
	$\left(=\frac{45}{60}\right)=\frac{3}{4}$ (*)	A1cso (3)
	(b) $\sin^2 A + \left(\frac{3}{4}\right)^2 = 1$ (or equiv. Pythag. method)	M1
	$\left(\sin^2 A = \frac{7}{16}\right)$ $\sin A = \frac{1}{4}\sqrt{7}$ or equivalent exact form, e.g. $\sqrt{\frac{7}{16}}$, $\sqrt{0.4375}$	A1 (2)
		5
	(a) M: Is also scored for $5^2 = 4^2 + 6^2 - (2\times4\times6\cos\theta)$ or $6^2 = 5^2 + 4^2 - (2\times5\times4\cos\theta)$ or $\cos\theta = \frac{4^2+6^2-5^2}{2\times4\times6}$ or $\cos\theta = \frac{5^2+4^2-6^2}{2\times5\times4}$. 1st A: Rearranged correctly and numerically correct (possibly unsimplified), in the form $\cos\theta = ...$ or $60\cos\theta = 45$ (or equiv. in the form $p\cos\theta = q$). <u>Alternative</u> (verification): $4^2 = 5^2 + 6^2 - \left(2\times5\times6\times\frac{3}{4}\right)$ [M1] Evaluate correctly, at least to $16 = 25 + 36 - 45$ [A1] Conclusion (perhaps as simple as a tick). [A1cso] (Just achieving 16 = 16 is insufficient without at least a tick). (b) M: Using a correct method to find an equation in $\sin^2 A$ or $\sin A$ which would give an exact value. <u>Correct answer without working</u> (or with unclear working or decimals): Still scores both marks.	

Q4.

Question Number	Scheme	Marks
	$\frac{\sin x}{16} = \frac{\sin 50^0}{13}$	M1
	$(\sin x) = \frac{16 \times \sin 50}{13}$ (= 0.943 but accept 0.94)	A1
	$x =$ awrt 70.5(3) **and** 109.5 **or** 70.6 and 109.4	dM1 A1 **(4)** **[4]**
	Notes M1: use sine formula correctly **in any form.** Allow awrt 0.77 for $\sin 50°$ A1: give the correct value or correct expression for $\sin x$ (this implies the M1 mark). If it is given as expression they do not need degrees symbol. $\frac{\sin 50 \times 16}{13}$ is fine, If this is given as a decimal allow answers which round to 0.94. Allow awrt −0.323 (radians) here but no further marks are available. If they give this as x (not sin$x$) and do not recover this is A0 dM1: Correct work leading to x=… (via inverse sin) expression or value for sin$x$ If the previous A mark has been awarded for a correct expression then this is for getting to awrt 70.5 or 109.5 (allow for 70.6 or 109.4). If the previous A mark was not gained, e.g. rounding errors were made in rearranging the correct sine formula then award dM1 for evidence of use of **inverse sin in degrees** on their value for sin$x$ (may need to check on calculator). **NB** 70.5 following a correct sine formula will gain M1A1M1. A1: deduce and state **both of the answers** x = 70.5 and 109.5 (do not need degrees) Accept awrt these. Also accept 70.6 and 109.4. (Second answer is sometimes obtained by a long indirect route but still scores A1) If working in radians throughout, answers are 1.23 and 1.91 and this can be awarded M1 A1 M1 A0 (Working with 50 radians gives probable answers of -0.3288 and 3.47 giving M1A1M0A0) **Special case:** Wrong labelling of triangle. This simplifies the problem as there is only one solution for angle x. So it is not treated as a misread. If they find the missing side as awrt 12.6 then proceed to find an angle or its sine or cosine then give M1A0M0A0	
	Alternative Method using cosine rule Let $BC = a$. M1: uses the cosine rule to form to form a three term quadratic equation in a (e.g. $a^2 - 32a\cos 50° + 87 = 0$ or $a^2 - \text{awrt}20.6a + 87 = 0$ though allow slips in signs rearranging) A1: Solves and obtains a correct value for a of awrt 14.6 **or** awrt 5.95. dM1: A **correct full** method to find (at least) one of the two angles. May use cosine rule again, or find angle BAC and then use sine rule. As in the main scheme, if the previous A mark has been awarded then they should obtain one of the correct angles for this mark. A1: deduces both correct answer as in main scheme. **NB** obtaining only one correct angle will usually score M1A1M1A0 in any method.	

Q5.

Question Number	Scheme		Marks
(a)	$\frac{1}{2}r^2\theta = \frac{1}{2}(6)^2\left(\frac{\pi}{3}\right) = 6\pi$ or 18.85 or awrt 18.8 $(\text{cm})^2$	Using $\frac{1}{2}r^2\theta$ (See notes) 6π or 18.85 or awrt 18.8	M1 A1 **[2]**
(b)	$\sin\left(\frac{\pi}{6}\right) = \frac{r}{6-r}$ $\frac{1}{2} = \frac{r}{6-r}$ $6 - r = 2r \Rightarrow r = 2$	$\sin\left(\frac{\pi}{6}\right)$ or $\sin 30^\circ = \frac{r}{6-r}$ Replaces sin by numeric value $r = 2$	M1 dM1 A1 **cso** **[3]**
(c)	Area $= 6\pi - \pi(2)^2 = 2\pi$ or awrt 6.3 $(\text{cm})^2$	their area of sector $- \pi r^2$ 2π or awrt 6.3	M1 A1 **cao** **[2]**

(a) M1: Needs θ in radians for this formula.
Candidate could convert to degrees and use the degrees formula.
A1: Does not need units. Answer should be either 6π or 18.85 or awrt 18.8
Correct answer with no working is M1A1.
This M1A1 can only be awarded in part (a).

(b) M1: Also allow $\cos\left(\frac{\pi}{3}\right)$ or $\cos 60^\circ = \frac{r}{6-r}$.

1st M1: Needs correct trigonometry method. Candidates could state $\sin\left(\frac{\pi}{6}\right) = \frac{r}{x}$ and $x + r = 6$ or equivalent in their working to gain this method mark.

dM1: Replaces sin by numerical value. $0.009... = \frac{r}{6-r}$ from working "incorrectly" in degrees is fine here for dM1.
A1: For $r = 2$ from correct solution only.

Alternative: 1st M1 for $\frac{r}{OC} = \sin 30$ or $\frac{r}{OC} = \cos 60$. 2nd M1 for $OC = 2r$ and then A1 for $r = 2$.

Note seeing $OC = 2r$ is M1M1.

Special Case: If a candidate states an answer of $r = 2$ (must be in part (b)) as a guess or from an incorrect method then award SC: M0M0B1. Such a candidate could then go on to score M1A1 in part (c).

(c) M1: For "their area of sector – their area of circle", where $r > 0$ is ft from their answer to part (b).
Allow the method mark if "their area of sector" < "their area of circle". The candidate must show somewhere in their working that they are subtracting the correct way round, even if their answer is negative.
Some candidates in part (c) will either use their value of r from part (b) or even introduce a value of r in part (c). You can apply the scheme to award either M0A0 or M1A0 or M1A1 to these candidates.
Note: Candidates can get M1 by writing "their part (a) answer $- \pi r^2$", where the radius of the circle is not substituted.
A1: cao – accept exact answer or awrt 6.3
Correct answer only with no working in (c) gets M1A1
Beware: The answer in (c) is the same as the arc length of the pendant