

Complex Numbers

Define $i = \sqrt{-1}$ an imaginary number

\mathbb{R} is the set of real numbers

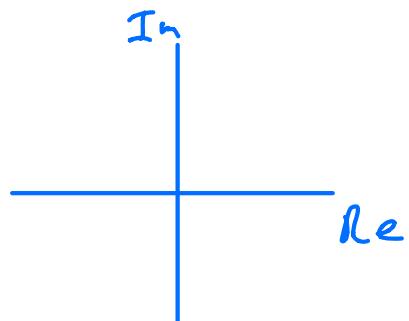
\mathbb{C} is the set of complex numbers of the form

$$a + bi$$

\uparrow \uparrow
real part imaginary part

where a, b are real

\mathbb{N}	natural numbers 1, 2, 3, 4, ...
\mathbb{Z}	integers
\mathbb{Q}	rationals
$\mathbb{I}\mathbb{R}$	irrationals
\mathbb{C}	complex



Arithmetic of Complex Numbers

Addition Let $z_1 = 3 + 4i$ $z_2 = 2 - 7i$

$$z_1 + z_2 = 3 + 4i + 2 - 7i = 5 - 3i$$

Subtraction

$$\begin{aligned} z_1 - z_2 &= (3 + 4i) - (2 - 7i) \\ &= 1 + 11i \end{aligned}$$

Multiplication

$$\begin{aligned}z_1 z_2 &= (3+4i)(2-7i) \\&= 6 + 8i - 21i - 28i^2 \\&= 6 + 8i - 21i + 28 \\&= 34 - 13i\end{aligned}$$

Division

$$\begin{aligned}\frac{z_2}{z_1} &= \frac{2-7i}{3+4i} \\&= \frac{(2-7i)}{(3+4i)} \times \frac{(3-4i)}{(3-4i)} \\&= \frac{6 - 21i - 8i - 28}{9 + 12i - 12i - 16i^2} \\&= \frac{-22 - 29i}{9 + 16} \\&= -\frac{22}{25} - \frac{29}{25}i\end{aligned}$$

Simplifying imaginary numbers

$$\sqrt{-9} = \sqrt{9 \times (-1)} = 3i$$

$$\sqrt{-49} = 7i$$

Exercise 4a

$$4 \text{ a) } \frac{4 - 2i}{\sqrt{2}} = 2\sqrt{2} - \sqrt{2}i$$

$$\begin{aligned} \text{b) } \frac{2 - 6i}{1 + \sqrt{3}} &= \frac{2 - 6i}{1 + \sqrt{3}} \times \frac{1 - \sqrt{3}}{1 - \sqrt{3}} \\ &= \frac{2 - 6i - 2\sqrt{3} + 6\sqrt{3}i}{1^2 - \sqrt{3}^2} \\ &= \frac{2 - 2\sqrt{3} + (6\sqrt{3} - 6)i}{-2} \\ &= \frac{-2 + 2\sqrt{3} - (6\sqrt{3} - 6)i}{2} \\ &= -1 + \sqrt{3} - (3\sqrt{3} - 3)i \\ \text{or } &\quad \sqrt{3} - 1 + (3 - 3\sqrt{3})i \end{aligned}$$

$$\text{c) } z_1 = a + 9i \quad z_2 = -3 + 5i$$

$$z_2 - z_1 = 7 + 2i$$

$$(-3 + 5i) - (a + 9i) = 7 + 2i$$

Equate real and imaginary parts Re and Im

$$-3 - a = 7$$

$$-3 - 7 = a$$

$$\underline{a = -10}$$

$$b - 9 = 2$$

$$b = 2 + 9$$

$$\underline{b = 11}$$

Solving Equations

A polynomial equation of degree n with real coefficients has precisely n complex roots

If $a + bi$ is a root of a polynomial then $a - bi$ is also a root

$a + bi$ and $a - bi$ are called
complex conjugates

Examples

$$1) \quad x^2 + 36 = 0$$

$$x^2 = -36$$

$$x = \pm \sqrt{-36}$$

$$x = \pm 6i$$

$$2) \quad x^2 + 5x + 10 = 0$$

$$x = \frac{-5 \pm \sqrt{25-40}}{2}$$

$$x = \frac{-5 \pm \sqrt{15}i}{2}$$

$$x = -\frac{5}{2} \pm \frac{\sqrt{15}i}{2}$$

Exercise 1B

$$1d \quad 3z^2 + 150 = 38 - z^2$$

$$4z^2 = -112$$

$$z^2 = -28$$

$$z = \pm \sqrt{-28}$$

$$z = \pm 2\sqrt{7}i$$

Exercise 1c

5) $(1+i)^6$

$$\begin{array}{r} 1 \\ | \\ 1 \ 2 \ 1 \\ | \ 3 \ 3 \ | \\ 1 \ 4 \ 6 \ 4 \ | \\ | \ 5 \ 10 \ 10 \ 5 \ | \\ 1 \ 6 \ 15 \ 20 \ 15 \ 6 \ 1 \end{array}$$

$$(a+b)^6 = a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$$

$$1^6 + 6(1^5i) + 15(1^4i^2) + 20(1^3i^3) + 15(1^2i^4) + 6(1i^5) + i^6$$

$$= 1 + 6i + 15i^2 + 20i^3 + 15i^4 + 6i^5 + i^6$$

$$= 1 + 6i - 15 - 20i + 15 + 6i - 1$$

$$= 0 - 8i$$

Homework

Exercise 1a Q7, Q8

Exercise 1b Q1f, Q2c, Q4a

Exercise 1c Q1i, Q4b, Q9