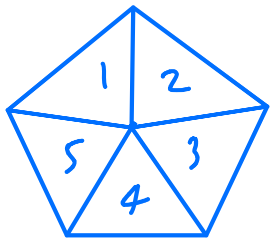
Binomial Distribution



Scoring 4 is a success

Any other score is a failure

On one spin

$P(\text{success}) = p = 0.2$

$P(\text{fail}) = 1 - p = q = 0.8$

Suppose spinner is spun 5 times

the $X \sim B(\overset{n}{5}, \overset{p}{0.2})$ X = number of successes

Manual Calculations S = Success
F = Fail

5 spins

$P(X=0)$ = FFFFF $P(X=0) = .8 \times .8 \times .8 \times .8 \times .8$

$= .8^5 = 0.3277$

$P(X=1)$ = SFFFF or FSFFF or FFSFF or FFFSF or FFFFS

$= .2 \times .8^4 \times 5C1 = 0.4096$

$P(X=2)$

SSFFF FSSFF FFSFS
SFSFF FSFSF FFFSS
SFFSF FSFFS
SFFFS FFSSF

$P(X=2) = {}^5C_2 \times 0.2^2 \times 0.8^3 = 0.2048$

In general if $X \sim B(n, p)$

then $P(X = r) = {}^nC_r \times p^r q^{n-r}$

or ${}^nC_r \times p^r (1-p)^{n-r}$

From Tables (Page 29)

$P(X = 0) = 0.3277$

$P(X = 1) = P(X \leq 1) - P(X = 0)$

$= 0.7373 - 0.3277 = 0.4096$

$P(X = 2) = P(X \leq 2) - P(X \leq 1) = 0.2048$

$0.9421 - 0.7373$

Example Let $X \sim B(\overset{n}{10}, \overset{p}{0.4})$

Find $P(4 \leq X \leq 7)$

$= P(X \leq 7) - P(X \leq 3)$

$= 0.9877 - 0.3822$

$= 0.6055$

6

8 At a doctor's surgery, records show that 20% of patients who make an appointment fail to turn up. During afternoon surgery the doctor has time to see 16 patients.

There are 16 appointments to see the doctor one afternoon.

(i) Find the probability that all 16 patients turn up. [2]

(ii) Find the probability that more than 3 patients do not turn up. [3]

$X \sim B(\overset{n}{16}, \overset{p}{0.2})$

X represents number not turning up

All turn up $P(X=0)$

By calc $= 0.0281$

Or $0.8^{16} = 0.0281$

ii) $P(X>3) = 1 - P(X \leq 3)$

$= 1 - 0.5981$

$= 0.4019$

5

7 A game requires 15 identical ordinary dice to be thrown in each turn.

Assuming the dice to be fair, find the following probabilities for any given turn.

(i) No sixes are thrown. [2]

(ii) Exactly four sixes are thrown. [3]

(iii) More than three sixes are thrown. [2]

5 Douglas plays darts, and the probability that he hits the number he is aiming at is 0.87 for any particular dart.

Douglas aims a set of three darts at the number 20; the number of times he is successful can be modelled by $B(3, 0.87)$.

(i) Calculate the probability that Douglas hits 20 twice. [3]

(ii) Douglas aims fifty sets of 3 darts at the number 20. Find the expected number of sets for which Douglas hits 20 twice. [1]

(iii) Douglas aims four sets of 3 darts at the number 20. Calculate the probability that he hits 20 twice for two sets out of the four. [2]

8 A multinational accountancy firm receives a large number of job applications from graduates each year. On average 20% of applicants are successful.

A researcher in the human resources department of the firm selects a random sample of 17 graduate applicants.

(i) Find the probability that at least 4 of the 17 applicants are successful. [3]

(ii) Find the expected number of successful applicants in the sample. [2]

(iii) Find the most likely number of successful applicants in the sample, justifying your answer. [3]

7 A particular product is made from human blood given by donors. The product is stored in bags. The production process is such that, on average, 5% of bags are faulty. Each bag is carefully tested before use.

(i) 12 bags are selected at random.

(A) Find the probability that exactly one bag is faulty. [3]

(B) Find the probability that at least two bags are faulty. [2]

(C) Find the expected number of faulty bags in the sample. [2]

(ii) A random sample of n bags is selected. The production manager wishes there to be a probability of one third or less of finding any faulty bags in the sample. Find the maximum possible value of n, showing your working clearly. [3]