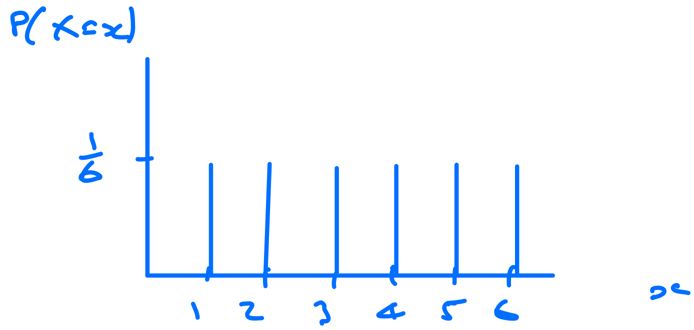
Probability Distributions

Uniform Distribution

Rolling a Dice

x	1	2	3	4	5	6
$P(X=x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$



Other Discrete Distributions

Spin coin 3 Times and count Heads

HHH
HHT
HTH
HTT
THH
THT
TTH
TTT

equally likely so each has prob $= \frac{1}{8}$

Let X be number of Heads

x	0	1	2	3
$P(X=x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Exercise 6A

1) a) height X cm not discrete because height is a continuous variable

b) number of 6's rolled is discrete countable - integer between 0 and 100

c number of days in week - discrete
countable integer 0, 1, 2, 3, 4, 5, 6, 7

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3)

2, 2
2, 3
3, 2
3, 3

x	4	5	6
X	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$

$$P(X=x) = \begin{cases} 0.25, & x = 4, 6 \\ 0.5, & x = 5 \end{cases}$$

probability mass function

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5) $P(X=x) = kx \qquad x = 1, 2, 3, 4$

a)

x	1	2	3	4
$P(X=x)$	k	$2k$	$3k$	$4k$

$$k + 2k + 3k + 4k = 1$$

$$10k = 1$$

$$k = \frac{1}{10}$$

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7)

$$P(X=x) = \begin{cases} 0.1 & x = -2, -1 \\ \beta & x = 0, 1 \\ 0.2 & x = 2 \end{cases}$$

x	-2	-1	0	1	2
$P(X=x)$	0.1	0.1	β	β	0.2

b) $0.1 + 0.1 + \beta + \beta + 0.2 = 1$

$2\beta + 0.4 = 1$

$\beta = 0.3$

x	-2	-1	0	1	2
$P(X=x)$	0.1	0.1	0.3	0.3	0.2

c $P(-1 \leq X < 2) = 0.1 + 0.3 + 0.3$

$= 0.7$

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9) a) $P(X=1) = \frac{1}{50}$

b) $P(X \geq 28) = \frac{50-27}{50} = \frac{23}{50}$

c) $P(13 < X < 42)$

$\frac{41-13}{50} = \frac{28}{50}$

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11) $P(H) = \frac{2}{3}$

T T T T $\quad P(TTTT) = \left(\frac{1}{3}\right)^4 = \frac{1}{81}$

T T T H $\quad P(TTTH) = \left(\frac{1}{3}\right)^3 \times \frac{2}{3} = \frac{2}{81}$

T T H $\quad P(TTH) = \left(\frac{1}{3}\right)^2 \times \frac{2}{3} = \frac{2}{27}$ or $\frac{6}{81}$

T H

H

$$P(TH) = \frac{1}{3} \times \frac{2}{3} = \frac{2}{9} = \frac{18}{81}$$

$$P(H) = \frac{2}{3} = \frac{54}{81}$$

X = number of tosses

x	1	2	3	4
$P(X=x)$	$\frac{54}{81}$	$\frac{18}{81}$	$\frac{6}{81}$	$\frac{3}{81}$

b) Find $P(X>2) = \frac{9}{81} = \frac{1}{9}$

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13) $P(X=x) = \frac{2}{x^2}$ $\quad x = 2, 3, 4$

$$P(X=2) + P(X=3) + P(X=4)$$

a) $$= \frac{2}{2^2} + \frac{2}{3^2} + \frac{2}{4^2}$$

$$= \frac{2}{4} + \frac{2}{9} + \frac{2}{16} = \frac{61}{72} \neq 1$$

Sum of probabilities of mutually exclusive outcomes do not sum to 1 so not a probability distribution.

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b) Need to multiply all probabilities by $\frac{72}{61}$

$$\frac{2\left(\frac{72}{61}\right)}{4} + \frac{2\left(\frac{72}{61}\right)}{9} + \frac{2\left(\frac{72}{61}\right)}{16} = 1$$

$$\therefore k = 2\left(\frac{72}{61}\right) = \frac{144}{61}$$

3 Jeremy is a computing consultant who sometimes works at home. The number, X, of days that Jeremy works at home in any given week is modelled by the probability distribution

$$P(X = r) = \frac{1}{40} r(r+1) \quad \text{for } r = 1, 2, 3, 4.$$

(i) Verify that $P(X = 4) = \frac{1}{2}$. [1]

~~**(ii)** Calculate $E(X)$ and $Var(X)$.~~ NOT ON SYLLABUS [5]

(iii) Jeremy works for 45 weeks each year. Find the expected number of weeks during which he works at home for exactly 2 days. [2]

i) $P(X=4) = \frac{1}{40} \times 4 \times (4+1)$

$= \frac{1}{40} \times 4 \times 5 = \frac{20}{40} = \frac{1}{2}$

iii) $P(X=2) = \frac{1}{40} \times 2 \times (2+1)$

$= \frac{6}{40} = \frac{3}{20}$

Expected number of weeks $= 45 \times \frac{3}{20}$

$= 6.75$ weeks

2 Four letters are taken out of their envelopes for signing. Unfortunately they are replaced randomly, one in each envelope.

The probability distribution for the number of letters, X, which are now in the correct envelope is given in the following table.

r	0	1	2	3	4
$P(X = r)$	$\frac{3}{8}$	$\frac{1}{3}$	$\frac{1}{4}$	0	$\frac{1}{24}$

(i) Explain why the case $X = 3$ is impossible. [1]

(ii) Explain why $P(X = 4) = \frac{1}{24}$. [2]

(iii) ~~Calculate $E(X)$ and $Var(X)$.~~ NOT ON SYLLABUS [5]

i) If 3 are correct so is the fourth.

Right Right Right Right

$P(4\text{ right}) = \frac{1}{4} \times \frac{1}{3} \times \frac{1}{2} \times 1 = \frac{1}{24}$