Surds

Surds are numbers such as $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ and expressions such as $3-2\sqrt{5}$, $1+\sqrt{3}$

Rules for Surds

Consider $(\sqrt{a} \times \sqrt{b}) \times (\sqrt{a} \times \sqrt{b})$

$$= \sqrt{a} \times \sqrt{b} \times \sqrt{a} \times \sqrt{b}$$

$$= \sqrt{a} \times \sqrt{a} \times \sqrt{b} \times \sqrt{b}$$

$$= a \times b$$

$$= ab$$

$$\therefore \quad \sqrt{a} \times \sqrt{b} = \sqrt{ab}$$

Consider $\dfrac{\sqrt{a}}{\sqrt{b}} \times \dfrac{\sqrt{a}}{\sqrt{b}} = \dfrac{\sqrt{a} \times \sqrt{a}}{\sqrt{b} \times \sqrt{b}} = \dfrac{a}{b}$

$$\therefore \quad \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

WARNING - there are no similar results for addition and subtraction

$$\sqrt{9} + \sqrt{4} \neq \sqrt{9+4} = \sqrt{13}$$

$$3 + 2 \neq 3.60555$$

$\sqrt{9} - \sqrt{4} \neq \sqrt{9-4} = \sqrt{5}$

$3 - 2 \neq 2.236$

EXERCISE 10J

1 Simplify each of the following. Leave your answers in surd form.

a $\sqrt{2} \times \sqrt{3}$ **b** $\sqrt{5} \times \sqrt{3}$ **c** $\sqrt{2} \times \sqrt{2}$ **d** $\sqrt{2} \times \sqrt{8} = 4$

e $\sqrt{5} \times \sqrt{8}$ **f** $\sqrt{3} \times \sqrt{3}$ **g** $\sqrt{6} \times \sqrt{2}$ **h** $\sqrt{7} \times \sqrt{3} = \sqrt{21}$

i $\sqrt{2} \times \sqrt{7}$ **j** $\sqrt{2} \times \sqrt{18}$ **k** $\sqrt{6} \times \sqrt{6}$ **l** $\sqrt{5} \times \sqrt{6} = \sqrt{30}$

2 Simplify each of the following. Leave your answers in surd form.

a $\sqrt{12} \div \sqrt{3}$ **b** $\sqrt{15} \div \sqrt{3}$ **c** $\sqrt{12} \div \sqrt{2}$ **d** $\sqrt{24} \div \sqrt{8} = \sqrt{3}$

e $\sqrt{40} \div \sqrt{8}$ **f** $\sqrt{3} \div \sqrt{3}$ **g** $\sqrt{6} \div \sqrt{2}$ **h** $\sqrt{21} \div \sqrt{3} = \sqrt{7}$

i $\sqrt{28} \div \sqrt{7}$ **j** $\sqrt{48} \div \sqrt{8}$ **k** $\sqrt{6} \div \sqrt{6}$ **l** $\sqrt{54} \div \sqrt{6} = 3$

3 Simplify each of the following. Leave your answers in surd form.

a $\sqrt{2} \times \sqrt{3} \times \sqrt{2}$ **b** $\sqrt{5} \times \sqrt{3} \times \sqrt{15}$ **c** $\sqrt{2} \times \sqrt{2} \times \sqrt{8}$ **d** $\sqrt{2} \times \sqrt{8} \times \sqrt{3} = \sqrt{48}$

e $\sqrt{5} \times \sqrt{8} \times \sqrt{8}$ **f** $\sqrt{3} \times \sqrt{3} \times \sqrt{3}$ **g** $\sqrt{6} \times \sqrt{2} \times \sqrt{48}$ **h** $\sqrt{7} \times \sqrt{3} \times \sqrt{3} = \sqrt{63}$

i $\sqrt{2} \times \sqrt{7} \times \sqrt{2}$ **j** $\sqrt{2} \times \sqrt{18} \times \sqrt{5}$ **k** $\sqrt{6} \times \sqrt{6} \times \sqrt{3}$ **l** $\sqrt{5} \times \sqrt{6} \times \sqrt{30} = 30$

1 a) $\sqrt{2} \times \sqrt{3} = \sqrt{2 \times 3} = \sqrt{6}$

1 e) $\sqrt{5} \times \sqrt{8} = \sqrt{5 \times 8} = \sqrt{40}$

1 i) $\sqrt{2} \times \sqrt{7} = \sqrt{2 \times 7} = \sqrt{14}$

2 a) $\sqrt{12} \div \sqrt{3} = \sqrt{\frac{12}{3}} = \sqrt{4} = 2$

2 e) $\sqrt{40} \div \sqrt{8} = \sqrt{\frac{40}{8}} = \sqrt{5}$

2 i) $\sqrt{28} \div \sqrt{7} = \sqrt{\frac{28}{7}} = \sqrt{4} = 2$

3 a) $\sqrt{2} \times \sqrt{3} \times \sqrt{2} = \sqrt{2 \times 3 \times 2} = \sqrt{12}$

3e) $\sqrt{5} \times \sqrt{8} \times \sqrt{8} = \sqrt{5 \times 8 \times 8} = \sqrt{320}$

3i) $\sqrt{2} \times \sqrt{7} \times \sqrt{2} = \sqrt{2 \times 7 \times 2} = \sqrt{28}$

Do the right hand column Q 1, 2, 3

We can improve on some of the answers above

3a) $\sqrt{2} \times \sqrt{3} \times \sqrt{2} = 2\sqrt{3}$

3e) $\sqrt{5} \times \sqrt{8} \times \sqrt{8} = 8\sqrt{5}$

3i) $\sqrt{2} \times \sqrt{7} \times \sqrt{2} = 2\sqrt{7}$

Perfect Squares

$1^2 = 1$ $2^2 = 4$ $3^2 = 9$ $4^2 = 16$ $5^2 = 25$

$6^2 = 36$ $7^2 = 49$ $8^2 = 64$ $9^2 = 81$ $10^2 = 100$

Simplifying Surds (only some can be simplified)

$$\sqrt{50} = \sqrt{25 \times 2} = \sqrt{25} \times \sqrt{2} = 5\sqrt{2}$$

Past exam question

Simplify $\sqrt{50} + \sqrt{2}$

$$= \sqrt{25 \times 2} + \sqrt{2}$$

$$= 5\sqrt{2} + \sqrt{2} = 6\sqrt{2}$$

---

6 Simplify each of the following surds into the form $a\sqrt{b}$.

a $\sqrt{18}$	**b** $\sqrt{24}$ $= 2\sqrt{6}$	**c** $\sqrt{12}$ $= 2\sqrt{3}$	**d** $\sqrt{50}$ $= 5\sqrt{2}$
e $\sqrt{8}$	**f** $\sqrt{27}$ $= 3\sqrt{3}$	**g** $\sqrt{48}$ $= 4\sqrt{3}$	**h** $\sqrt{75}$ $= 5\sqrt{3}$
i $\sqrt{45}$	**j** $\sqrt{63}$ $= 3\sqrt{7}$	**k** $\sqrt{32}$ $= 4\sqrt{2}$	**l** $\sqrt{200}$ $= 10\sqrt{2}$
m $\sqrt{1000}$	**n** $\sqrt{250}$ $= 5\sqrt{10}$	**o** $\sqrt{98}$ $= 7\sqrt{2}$	**p** $\sqrt{243}$ $= 9\sqrt{3}$

6 a) $\sqrt{18} = \sqrt{9 \times 2} = 3\sqrt{2}$

6 e) $\sqrt{8} = \sqrt{4 \times 2} = 2\sqrt{2}$

6 i) $\sqrt{45} = \sqrt{9 \times 5} = 3\sqrt{5}$

6 m) $\sqrt{1000} = \sqrt{100 \times 10} = 10\sqrt{10}$

Do the rest of Q6

Typical Exam Questions

1) Simplify $\sqrt{48} + \sqrt{27}$

$= \sqrt{16 \times 3} + \sqrt{9 \times 3} = 4\sqrt{3} + 3\sqrt{3} = 7\sqrt{3}$

2) Simplify $\sqrt{50} - \sqrt{8}$

$= \sqrt{25 \times 2} - \sqrt{4 \times 2} = 5\sqrt{2} - 2\sqrt{2} = 3\sqrt{2}$

3) Simplify $\sqrt{20} + \sqrt{45} - \sqrt{5}$

$= \sqrt{4 \times 5} + \sqrt{9 \times 5} - \sqrt{5}$

$= 2\sqrt{5} + 3\sqrt{5} - \sqrt{5} = 4\sqrt{5}$
