Surds Exercise 13.3A Page 273

Exercise 13.3A

1 You are told that $\sqrt{2} \approx 1.414$, $\sqrt{3} \approx 1.732$ and $\sqrt{5} \approx 2.236$. Use this information to estimate the value of each of these. Show your working, and give your answers to 4 significant figures.

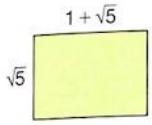
a $\sqrt{2} + \sqrt{3}$ b $\sqrt{10}$

c $\sqrt{125}$ d $\sqrt{120}$

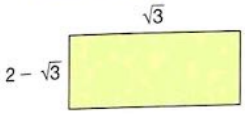
2 Write down four more surds whose value can be estimated using the information given in question **1**, and estimate the values of your surds.

3 Find the perimeter and area of each shape. Give your answers in simplified surd form.

a



b



4 Expand the brackets and simplify.

a $(1 + \sqrt{5})(2 + \sqrt{5})$

b $(\sqrt{3} - 1)(2 + \sqrt{3})$

c $(4 - \sqrt{7})(6 - 2\sqrt{7})$

d $(5 + \sqrt{2})(5 - \sqrt{2})$

e $(6 + 2\sqrt{5})(6 - 2\sqrt{5})$

f $(7 + 5\sqrt{3})(7 - 5\sqrt{3})$

5

$\sqrt{5}$	$2 - \sqrt{5}$	$\sqrt{64}$	$8\sqrt{2}$
$\sqrt{2}$	$\sqrt{20}$	$3 + \sqrt{5}$	$2 + \sqrt{5}$
$3 - \sqrt{5}$	$3\sqrt{2}$	$\sqrt{10}$	$\sqrt{100}$

From this table, find

a two numbers which are rational

b a pair which add up to give 4

c a pair where one number is double the other

d a pair where one is the other squared

e a pair which multiply to give 4

f a pair which multiply to give 48

g the largest value

h the smallest value.

6 Rewrite these fractions without surds in the denominators.

a $\dfrac{1}{1 - 2\sqrt{3}}$ b $\dfrac{\sqrt{5}}{1 + \sqrt{5}}$

c $\dfrac{1 - \sqrt{2}}{1 + \sqrt{2}}$ d $\dfrac{4 + \sqrt{2}}{3 - 2\sqrt{2}}$

7 Simplify these expressions.

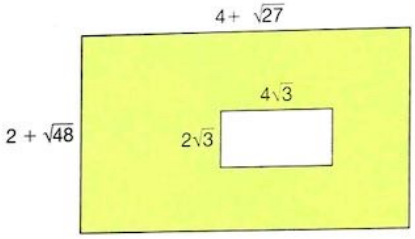
a $\dfrac{6 + \sqrt{27}}{3} - \dfrac{2 + \sqrt{3}}{4}$

b $\dfrac{20 + 3\sqrt{7}}{3} - (5 + \sqrt{28})$

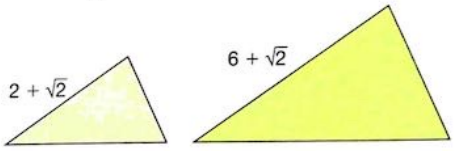
c $\dfrac{6 + \sqrt{8}}{2} + \dfrac{3 + \sqrt{2}}{9}$

d $\dfrac{8 + \sqrt{45}}{3} - \dfrac{4 + \sqrt{20}}{9}$

8 Find the size of the shaded area. Give your answer in the form $p(1 + \sqrt{3})$ where p is an integer.



9 The diagram shows two similar triangles.



Find the scale factor of enlargement. Give your answer in simplified surd form.

10 The numbers X, Y and Z are such that $Y^2 = XZ$.

a Find the number Z if $X = \sqrt{3}$ and $Y = 1 - \sqrt{3}$. Give your answer in the form $p + q\sqrt{3}$.

b Explain why it would not be possible to find Y if $X = \sqrt{5}$ and $Z = 1 - \sqrt{5}$.

1064, 1065 SEARCH

5

$\sqrt{5}$	$2-\sqrt{5}$	$\sqrt{64}$	$8\sqrt{2}$
$\sqrt{2}$	$\sqrt{20}$	$3+\sqrt{5}$	$2+\sqrt{5}$
$3-\sqrt{5}$	$3\sqrt{2}$	$\sqrt{10}$	$\sqrt{100}$

rational if can be written as $\frac{p}{q}$ where p, q are integers

From this table, find

a two numbers which are rational $\sqrt{100}$ $\sqrt{64}$

b a pair which add up to give 4 $2+\sqrt{5}$ $2-\sqrt{5}$

c a pair where one number is double the other $\sqrt{5}$ and $\sqrt{20} = 2\sqrt{5}$

d a pair where one is the other squared $\sqrt{10}$ and $\sqrt{100}$

e a pair which multiply to give 4 $3+\sqrt{5}$, $3-\sqrt{5}$

f a pair which multiply to give 48 $8\sqrt{2}$ and $3\sqrt{2}$

g the largest value $8\sqrt{2}$

h the smallest value. $2-\sqrt{5}$

6) d $\frac{4+\sqrt{2}}{3-2\sqrt{2}}$ $= \frac{4+\sqrt{2}}{3-2\sqrt{2}} \times \frac{3+2\sqrt{2}}{3+2\sqrt{2}}$

$= \frac{12+3\sqrt{2}+8\sqrt{2}+4}{3^2-(2\sqrt{2})^2}$

$= \frac{16+11\sqrt{2}}{9-8} = 16+11\sqrt{2}$

Exercise Complete 6 a, b, c

6 Rewrite these fractions without surds in the denominators.

a $\dfrac{1}{1-2\sqrt{3}}$ **b** $\dfrac{\sqrt{5}}{1+\sqrt{5}}$

c $\dfrac{1-\sqrt{2}}{1+\sqrt{2}}$ **d** $\dfrac{4+\sqrt{2}}{3-2\sqrt{2}}$

6a) $\dfrac{1}{1-2\sqrt{3}} = \dfrac{1}{1-2\sqrt{3}} \times \dfrac{1+2\sqrt{3}}{1+2\sqrt{3}}$

$= \dfrac{1+2\sqrt{3}}{1^2-(2\sqrt{3})^2} = \dfrac{1+2\sqrt{3}}{1-12}$

$= \dfrac{1+2\sqrt{3}}{-11}$

or $\dfrac{-1-2\sqrt{3}}{11}$

6b) $\dfrac{\sqrt{5}}{1+\sqrt{5}} = \dfrac{\sqrt{5}}{1+\sqrt{5}} \times \dfrac{1-\sqrt{5}}{1-\sqrt{5}} = \dfrac{\sqrt{5}-5}{1^2-\sqrt{5}^2}$

$= \dfrac{\sqrt{5}-5}{1-5}$

$= \dfrac{\sqrt{5}-5}{-4}$

or $\dfrac{5-\sqrt{5}}{4}$

6c)

$$\frac{1-\sqrt{2}}{1+\sqrt{2}} = \frac{1-\sqrt{2}}{1+\sqrt{2}} \times \frac{1-\sqrt{2}}{1-\sqrt{2}} = \frac{1-\sqrt{2}-\sqrt{2}+2}{1^2-\sqrt{2}^2}$$

$$= \frac{3-2\sqrt{2}}{1-2}$$

$$= \frac{3-2\sqrt{2}}{-1}$$

$$= 2\sqrt{2}-3$$

---

7 Simplify these expressions.

a $\dfrac{6+\sqrt{27}}{3} - \dfrac{(2+\sqrt{3})}{4}$

b $\dfrac{20+3\sqrt{7}}{3} - (5+\sqrt{28})$

c $\dfrac{6+\sqrt{8}}{2} + \dfrac{3+\sqrt{2}}{9}$

d $\dfrac{8+\sqrt{45}}{3} - \dfrac{4+\sqrt{20}}{9}$

7b)

$$\frac{20+3\sqrt{7}}{3} - (5+\sqrt{28})$$

$$= \frac{20+3\sqrt{7}}{3} - (5+2\sqrt{7})$$

$$= \frac{20+3\sqrt{7} - 3(5+2\sqrt{7})}{3}$$

$$= \frac{20+3\sqrt{7}-15-6\sqrt{7}}{3} = \frac{5-3\sqrt{7}}{3}$$

7a) $\frac{(6+\sqrt{27})}{3} - \frac{(2+\sqrt{3})}{4}$

$= \frac{4(6+\sqrt{27}) - 3(2+\sqrt{3})}{12}$

$= \frac{4(6+3\sqrt{3}) - 3(2+\sqrt{3})}{12}$

$= \frac{24+12\sqrt{3}-6-3\sqrt{3}}{12}$

$= \frac{18+9\sqrt{3}}{12} = \frac{9(2+\sqrt{3})}{12}$

$= \frac{3(2+\sqrt{3})}{4}$

Classwork 7c and 7d

7c $\frac{6+\sqrt{8}}{2} + \frac{3+\sqrt{2}}{9} = \frac{9(6+2\sqrt{2}) + 2(3+\sqrt{2})}{18}$

$= \frac{54+18\sqrt{2}+6+2\sqrt{2}}{18}$

$= \frac{60+20\sqrt{2}}{18}$

$= \frac{30+10\sqrt{2}}{9}$

7 **d** $\dfrac{8+\sqrt{45}}{3} - \dfrac{4+\sqrt{20}}{9}$ $= \dfrac{3(8+3\sqrt{5})-(4+2\sqrt{5})}{9}$

$= \dfrac{24+9\sqrt{5}-4-2\sqrt{5}}{9}$

$= \dfrac{20+7\sqrt{5}}{9}$