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U61M

Year 13 A-Level Mathematics

Mock Pure Paper 100 Marks

2 Hours

January 2019

1. There were 2100 tonnes of wheat harvested on a farm during 2017.

The mass of wheat harvested during each subsequent year is expected to increase by 1.2% per year.

(a) Find the total mass of wheat expected to be harvested from 2017 to 2030 inclusive, giving your answer to 3 significant figures.

(2)

Each year it costs

- £5.15 per tonne to harvest the first 2000 tonnes of wheat
- £6.45 per tonne to harvest wheat in excess of 2000 tonnes

(b) Use this information to find the expected cost of harvesting the wheat from 2017 to 2030 inclusive. Give your answer to the nearest £1000

(3)

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Question 1 continued

(Total for Question 1 is 5 marks)

2. The curve C has equation

$$y = 2x^3 + 5$$

The curve C passes through the point $P(1, 7)$.

Use differentiation from first principles to find the value of the gradient of the tangent to C at P.

(5)

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Question 2 continued

(Total for Question 2 is 5 marks)

3. The function f is defined by

$$\mathrm{f}: x \mapsto \frac{3x-5}{x+1}, \quad x \in \mathbb{R}, \ x \neq -1$$

(a) Find $\mathrm{f}^{-1}(x)$.

(3)

(b) Show that

$$\mathrm{ff}(x) = \frac{x+a}{x-1}, \quad x \in \mathbb{R}, \ x \neq \pm 1$$

where a is an integer to be found.

(4)

The function g is defined by

$$\mathrm{g}: x \mapsto x^2 - 3x, \quad x \in \mathbb{R}, 0 \leqslant x \leqslant 5$$

(c) Find the value of $\mathrm{fg}(2)$.

(2)

(d) Find the range of g.

(3)

(e) Explain why the function g does not have an inverse.

(1)

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Question 3 continued

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Question 3 continued

(Total for Question 3 is 13 marks)

4.

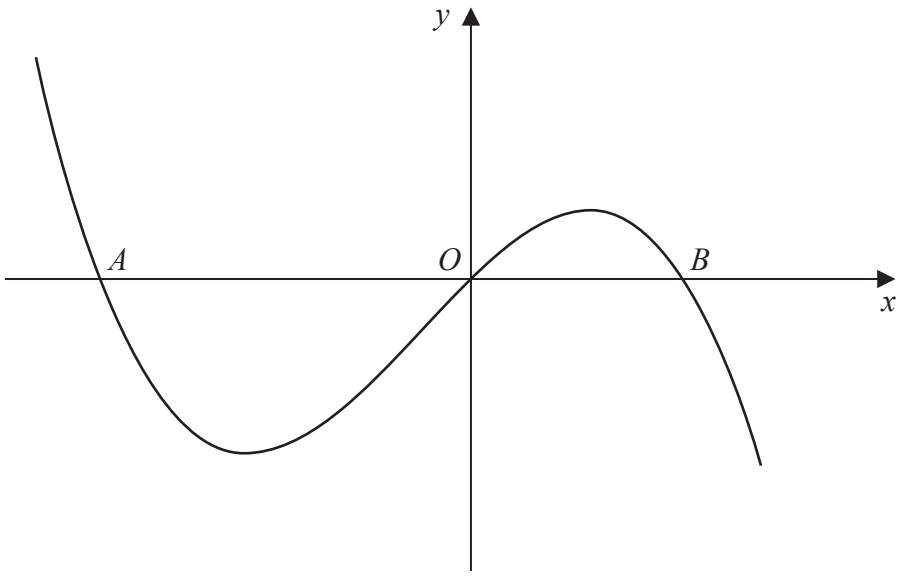

Figure 5

Figure 5 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$.
The curve C crosses the x-axis at the origin, O, and at the points A and B as shown in Figure 5.

Given that

$$\mathrm{f}'(x) = k - 4x - 3x^2$$

where k is constant,

(a) show that C has a point of inflection at $x = -\dfrac{2}{3}$ **(3)**

Given also that the distance $AB = 4\sqrt{2}$

(b) find, showing your working, the integer value of k. **(7)**

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Question 4 continued

(Total for Question 4 is 10 marks)

5. (a) Express $2\sin\theta - 1.5\cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

State the value of R and give the value of α to 4 decimal places.

(3)

Tom models the depth of water, D metres, at Southview harbour on 18th October 2017 by the formula

$$D = 6 + 2\sin\left(\frac{4\pi t}{25}\right) - 1.5\cos\left(\frac{4\pi t}{25}\right), \qquad 0 \leqslant t \leqslant 24$$

where t is the time, in hours, after 00:00 hours on 18th October 2017.

Use Tom's model to

(b) find the depth of water at 00:00 hours on 18th October 2017,

(1)

(c) find the maximum depth of water,

(1)

(d) find the time, in the afternoon, when the maximum depth of water occurs. Give your answer to the nearest minute.

(3)

Tom's model is supported by measurements of D taken at regular intervals on 18th October 2017. Jolene attempts to use a similar model in order to model the depth of water at Southview harbour on 19th October 2017.

Jolene models the depth of water, H metres, at Southview harbour on 19th October 2017 by the formula

$$H = 6 + 2\sin\left(\frac{4\pi x}{25}\right) - 1.5\cos\left(\frac{4\pi x}{25}\right), \qquad 0 \leqslant x \leqslant 24$$

where x is the time, in hours, after 00:00 hours on 19th October 2017.

By considering the depth of water at 00:00 hours on 19th October 2017 for both models,

(e) (i) explain why Jolene's model is not correct,

(ii) hence find a suitable model for H in terms of x.

(3)

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Question 5 continued

Question 5 continued

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Question 5 continued

(Total for Question 5 is 11 marks)

6.

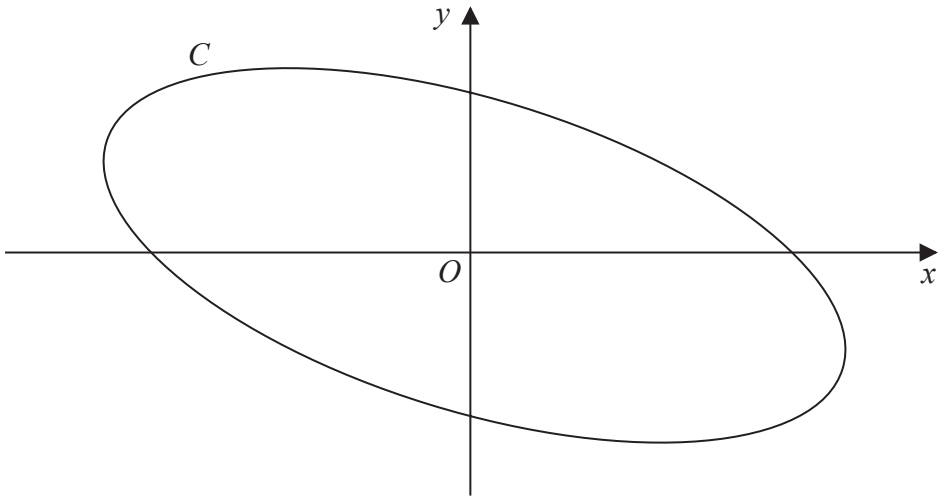

Figure 6

Figure 6 shows a sketch of the curve C with parametric equations

$$x = 4\cos\left(t + \frac{\pi}{6}\right),\ y = 2\sin t, \qquad 0 < t \leqslant 2\pi$$

Show that a Cartesian equation of C can be written in the form

$$(x + y)^2 + ay^2 = b$$

where a and b are integers to be found.

(5)

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Question 6 continued

Question 6 continued

(Total for Question 6 is 5 marks)

7. (a) Sketch the graph with equation

$$y = |2x - 5|$$

stating the coordinates of any points where the graph cuts or meets the coordinate axes.

(2)

(b) Find the values of x which satisfy

$$|2x - 5| > 7$$

(2)

(c) Find the values of x which satisfy

$$|2x - 5| > x - \frac{5}{2}$$

Write your answer in set notation.

(2)

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Question 7 continued

(Total for Question 7 is 6 marks)

8. The line l has equation

$$3x - 2y = k$$

where k is a real constant.

Given that the line l intersects the curve with equation

$$y = 2x^2 - 5$$

at two distinct points, find the range of possible values for k.

(5)

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Question 8 continued

(Total for Question 8 is 5 marks)

9.

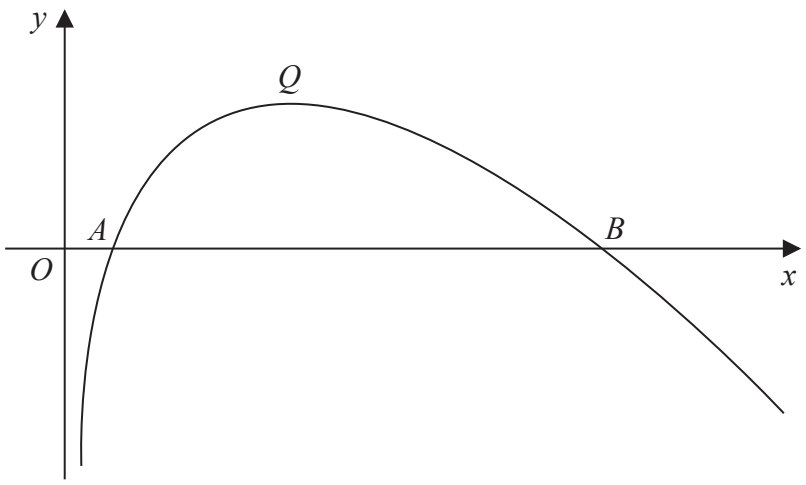

Figure 2

Figure 2 shows a sketch of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = (8 - x)\ln x, \qquad x > 0$$

The curve cuts the x-axis at the points A and B and has a maximum turning point at Q, as shown in Figure 2.

(a) Find the x coordinate of A and the x coordinate of B.

(1)

(b) Show that the x coordinate of Q satisfies

$$x = \frac{8}{1 + \ln x}$$

(4)

(c) Show that the x coordinate of Q lies between 3.5 and 3.6

(2)

(d) Use the iterative formula

$$x_{n+1} = \frac{8}{1 + \ln x_n} \qquad n \in \mathbb{N}$$

with $x_1 = 3.5$ to

(i) find the value of x_5 to 4 decimal places,

(ii) find the x coordinate of Q accurate to 2 decimal places.

(2)

Question 9 continued

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Question 9 continued

(Total for Question 9 is 9 marks)

10. A circle with centre $A(3, -1)$ passes through the point $P(-9, 8)$ and the point $Q(15, -10)$

(a) Show that PQ is a diameter of the circle.

(2)

(b) Find an equation for the circle.

(3)

A point R also lies on the circle.
Given that the length of the chord PR is 20 units,

(c) find the length of the shortest distance from A to the chord PR.
Give your answer as a surd in its simplest form.

(2)

(d) Find the size of angle ARQ, giving your answer to the nearest 0.1 of a degree.

(2)

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Question 10 continued

(Total for Question 10 is 9 marks)

11. The second, third and fourth terms of an arithmetic sequence are $2k$, $5k - 10$ and $7k - 14$ respectively, where k is a constant.

Show that the sum of the first n terms of the sequence is a square number.

(5)

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Question 11 continued

(Total for Question 11 is 5 marks)

12. A curve C is given by the equation

$$\sin x + \cos y = 0.5 \qquad -\frac{\pi}{2} \leqslant x < \frac{3\pi}{2}, -\pi < y < \pi$$

A point P lies on C.

The tangent to C at the point P is parallel to the x-axis.

Find the exact coordinates of all possible points P, justifying your answer.
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(7)

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Question 12 continued

(Total for Question 12 is 7 marks)

13. (a) Show that

$$\operatorname{cosec} 2x + \cot 2x \equiv \cot x, \qquad x \neq 90n°,\ n \in \mathbb{Z}$$

(5)

(b) Hence, or otherwise, solve, for $0 \leqslant \theta < 180°$,

$$\operatorname{cosec}(4\theta + 10°) + \cot(4\theta + 10°) = \sqrt{3}$$

You must show your working.
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

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Question 13 continued

(Total for Question 13 is 10 marks)