Spheres

$$\text{Volume} = \frac{4}{3}\pi r^3$$

$$\text{Surface Area} = 4\pi r^2$$

Ex1

Find volume and surface area of sphere radius 8cm

$$\text{Vol} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times 8^3 = 2145\,cm^3$$

$$\text{Surface Area} = 4\pi r^2 = = 4 \times \pi \times 8^2 = 804\,cm^2$$

Ex2 A sphere has volume $600\,cm^3$ find its surface area

$$V = \frac{4}{3}\pi r^3$$

$$3V = 4\pi r^3$$

$$\frac{3V}{4\pi} = r^3$$

$$\sqrt[3]{\frac{3V}{4\pi}} = r$$

$$r = \sqrt[3]{\frac{3 \times 600}{4\pi}}$$

$$r = 5.232$$

$$\text{Surface Area} = 4\pi r^2$$

$$= 4\pi \times 5.232^2$$

$$= 344\,cm^2$$

Ex3 A quarter of a sphere of radius 10cm Find volume and total surface area

$$\text{Vol} = \frac{\frac{4}{3}\pi r^3}{4} = \frac{1}{3}\pi r^3$$

$$= \frac{1}{3}\pi \times 10^3$$

$$= 1047 \text{ cm}^3$$

Total surface area

$= \frac{1}{4}$ surface area of sphere + 2 semi-circles radius 10 cm

$$= \frac{4\pi r^2}{4} + 2 \times \frac{\pi r^2}{2} = 2\pi r^2$$

$$= 2\pi \times 10^2$$

$$= 628 \text{ cm}^2$$

EXERCISE 4H

A

1. Calculate the volume of each of these spheres. Give your answers in terms of π.
   a Radius 3 cm b Radius 6 cm c Diameter 20 cm
2. Calculate the surface area of each of these spheres. Give your answers in terms of π.
   a Radius 3 cm b Radius 5 cm c Diameter 14 cm
3. Calculate the volume and the surface area of a sphere with a diameter of 50 cm.
4. A sphere fits exactly into an open cubical box of side 25 cm. Calculate the following.
   a the surface area of the sphere b the volume of the sphere
5. A metal sphere of radius 15 cm is melted down and recast into a solid cylinder of radius 6 cm. Calculate the height of the cylinder.
6. Lead has a density of 11.35 g/cm^3. Calculate the maximum number of shot (spherical lead pellets) of radius 1.5 mm which can be made from 1 kg of lead.
7. Calculate, correct to one decimal place, the radius of a sphere
   a whose surface area is 150 cm^2 b whose volume is 150 cm^3.

1. a) $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times 3^3 = 36\pi \text{ cm}^3$

b) $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times 6^3 = 288\pi \text{ cm}^3$

$$\begin{array}{r} 36 \\ \underline{48} \\ 288 \end{array}$$

c) $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times 10^3 = \frac{4000\pi}{3} \text{ cm}^3$

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2. a) $S.A. = 4\pi r^2 = 4\pi \times 3^2 = 36\pi \text{ cm}^2$

b) $S.A = 4\pi r^2 = 4\pi \times 5^2 = 100\pi \text{ cm}^2$

c) $S.A = 4\pi r^2 = 4\pi \times 7^2 = 196\pi \text{ cm}^2$

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3. Diameter 50 cm $\Rightarrow$ $r = 25$ cm

$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi \times 25^3$

$V = 65450 \text{ cm}^3$

$S.A. = 4\pi r^2$

$= 4\pi \times 25^2$

$= 7854 \text{ cm}^2$

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4. Fits into cube side 25 cm so $r = 12.5$ cm

$V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi \times 12.5^3$

$= 8181 \text{ cm}^3$

$S.A. = 4\pi r^2$

$= 4\pi \times 12.5^2$

$= 1963 \text{ cm}^2$

5.

5 A metal sphere of radius 15 cm is melted down and recast into a solid cylinder of radius 6 cm. Calculate the height of the cylinder.

Find volume of metal

$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times 15^3 = 4500\pi \text{ cm}^3$

Cylinder

$V = \pi r^2 h$

$\frac{V}{\pi r^2} = h$

$h = \frac{4500\cancel{\pi}}{\cancel{\pi} \times 6^2}$

$h = \frac{4500}{36}$

$h = 125 \text{ cm}$

6 Lead has a density of 11.35 g/cm^3. Calculate the maximum number of shot (spherical lead pellets) of radius 1.5 mm which can be made from 1 kg of lead.

radius = 0.15 cm

Mass of pellet = Density × Volume

$= 11.35 \times \frac{4}{3}\pi \times 0.15^3 \text{ g}$

$= 0.1604568448 \text{ g}$

Number made $= \frac{1000}{0.1604568448} = 6232.20531$

so 6232 pellets can be made.

7 Calculate, correct to one decimal place, the radius of a sphere

a whose surface area is 150 cm^2

b whose volume is 150 cm^3.

a)

$S.A = 150\ cm^2$

$4\pi r^2 = 150$

$r^2 = \frac{150}{4\pi}$

$r = \sqrt{\frac{150}{4\pi}}$

$r = 3.454$

$r = 3.5\ cm$ to 1 d.p.

$V = \frac{4}{3}\pi r^3$

$3V = 4\pi r^3$

$\frac{3V}{4\pi} = r^3$

$\sqrt[3]{\frac{3V}{4\pi}} = r$

$\sqrt[3]{\frac{3 \times 150}{4\pi}} = r$

$r = 3.30\ cm$

$r = 3.3\ cm$ to 1 d.p.

Frustums

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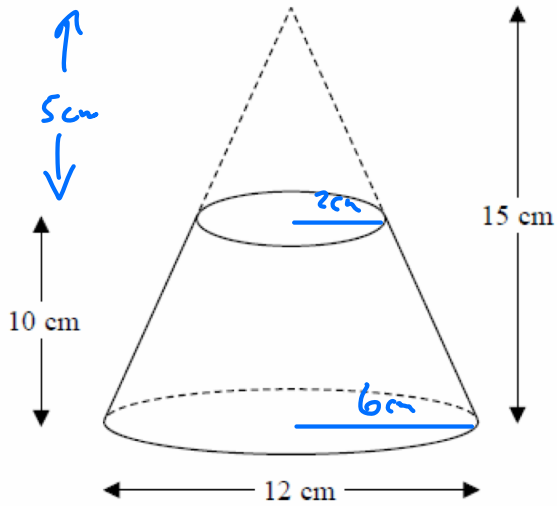
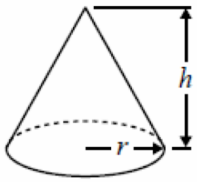
Q4.

A frustum is made by removing a small cone from a large cone as shown in the diagram.



Volume of cone $= \frac{1}{3}\pi r^2 h$



The frustum is made from glass.
The glass has a density of 2.5 g / cm^3

Work out the mass of the frustum.
Give your answer to an appropriate degree of accuracy.

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Vol large cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \times 6^2 \times 15$

Vol small cone $= \frac{1}{3}\pi \times 2^2 \times 5$

Vol of frustum $= \frac{1}{3}\pi(36 \times 15 - 4 \times 5)$

$= \frac{520\pi}{3}$

Mass $= \frac{520\pi}{3} \times 2.5 = 1361.36$ g

$= 1360\,g$