

MacLaurin Series Expansion

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \dots$$

Suppose $f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \dots$ an infinite expansion and that the function is infinitely differentiable

$$f(0) = a_0$$

$$f'(x) = a_1 + 2a_2x + 3a_3x^2 + \dots$$

$$f'(0) = a_1$$

$$f''(x) = 2a_2 + 3 \cdot 2 a_3 x + \dots$$

$$f''(0) = 2a_2 \Rightarrow a_2 = \frac{f''(0)}{2!}$$

$$f'''(x) = 3 \cdot 2 a_3 +$$

$$f'''(0) = 3! a_3 \Rightarrow a_3 = \frac{f'''(0)}{3!}$$

Example $f(x) = e^x \quad f(0) = e^0 = 1$
 $f'(x) = e^x \quad f'(0) = e^0 = 1$

$$f''(x) = e^x \quad f''(0) = e^0 = 1$$

$$\Rightarrow f(x) = e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} +$$

E x 2	$f(x) = \sin x$	$f(0) = \sin 0 = 0$
	$f'(x) = \cos x$	$f'(0) = \cos 0 = 1$
	$f''(x) = -\sin x$	$f''(0) = -\sin 0 = 0$
	$f'''(x) = -\cos x$	$f'''(0) = -\cos 0 = -1$
	$f^{(4)}(x) = \sin x$	$f^{(4)}(0) = \sin 0 = 0$

$$f(x) = \sin x = f(0) + f'(0)x + f''(0)\frac{x^2}{2!} + f'''(0)\frac{x^3}{3!} +$$

$$= 0 + x + \frac{0x^2}{2!} - \frac{1x^3}{3!} + \frac{0x^4}{4!} + \frac{1x^5}{5!} +$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} +$$

$f(x) = \cos x$	$f(x) = \cos x$	$f(0) = \cos 0 = 1$
	$f'(x) = -\sin x$	$f'(0) = -\sin 0 = 0$
	$f''(x) = -\cos x$	$f''(0) = -\cos 0 = -1$
	$f'''(x) = \sin x$	$f'''(0) = \sin 0 = 0$
	$f^{(4)}(x) = \cos x$	$f^{(4)}(0) = \cos 0 = 1$

$$f(x) = \cos x = 1 + 0x - \frac{1x^2}{2!} + \frac{0x^3}{3!} + \frac{1x^4}{4!} - +$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - +$$

$$f(x) = \ln(1+x) \quad f'(x) = \ln(1+x) \quad f(0) = 0$$

$$f'(x) = \frac{1}{1+x} \quad f'(0) = 1$$

$$(1+x)^{-1} \quad f''(x) = -\frac{1}{(1+x)^2} \quad f''(0) = -1$$

$$-1(1+x)^{-2} \times 1 \quad f'''(x) = \frac{2}{(1+x)^3} \quad f'''(0) = 2$$

$$-(1+x)^{-2} \quad f^{(iv)}(x) = -\frac{6}{(1+x)^4} \quad f^{(iv)}(0) = -6$$

$$2(1+x)^{-3} \quad \ln(1+x) = f(0) + f'(0)x + f''(0)\frac{x^2}{2!}$$

$$+ f'''(0)\frac{x^3}{3!} + f^{(iv)}(0)\frac{x^4}{4!} + +$$

$$\ln(1+x) = 0 + 1x + -\frac{1x^2}{2!} + \frac{2x^3}{3!} - \frac{6x^4}{4!} + -$$

$$= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \frac{x^6}{6} + -$$

Exercise 2C

$$1(b) \quad f(x) = \sqrt{1+x} = (1+x)^{\frac{1}{2}} \quad f(0) = 1$$

$$f'(x) = \frac{1}{2}(1+x)^{-\frac{1}{2}} \times 1 \quad f'(0) = \frac{1}{2}$$

$$f''(x) = -\frac{1}{4}(1+x)^{-\frac{3}{2}} \times 1 \quad f''(0) = -\frac{1}{4}$$

$$f'''(x) = \frac{3}{8}(1+x)^{-\frac{5}{2}} \times 1 \quad f'''(0) = \frac{3}{8}$$

$$f^{(4)}(x) = -\frac{15}{16}(1+x)^{-\frac{7}{2}} \times 1 \quad f^{(4)}(0) = -\frac{15}{16}$$

$$f(x) = \sqrt{1+x} = 1 + \frac{1}{2}x - \frac{1}{4}\frac{x^2}{2!} + \frac{3}{8}\frac{x^3}{3!} - \frac{15}{16}\frac{x^4}{4!}$$

$$= 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 - \frac{15}{384}x^4 +$$

$$f(x) = \tan x \quad f(0) = 0$$

$$f'(x) = \frac{1}{\cos^2 x} = (\cos x)^{-2} \quad f'(0) = 1$$

$$\begin{aligned} f''(x) &= -2(\cos x)^{-3}(-\sin x) & f''(0) &= 0 \\ &= +2 \sin x (\cos x)^{-3} \end{aligned}$$

$$\begin{aligned} f'''(x) &= 2 \sin x \times -3(\cos x)^{-4}(-\sin x) + (\cos x)^{-3} \times 2 \cos x \\ &= \frac{-6 \sin^2 x}{\cos^4 x} + \frac{2}{\cos^2 x} & f'''(0) &= 2 \end{aligned}$$

$$\begin{aligned}\tan x &= 0 + 1x + 0 \frac{x^2}{2!} + 2 \frac{x^3}{3!} + \\&= x + \frac{x^3}{3} +\end{aligned}$$
