Sine Role and Cosine Rule - Which One?

To use sine rule or cosine rule to 'solve' a triangle we need to know:

3 sides
or 2 sides and 1 angle
or 2 angles and 1 side

If one set of the above criteria are known it is possible to find all 3 sides and all 3 angles of the triangle.

It is not enough to know 3 angles because the triangle can be any size as it can be enlarged preserving the angles.

Sine rule requires complete knowledge of an opposite pair (angle and opposite side)

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine rule is used when two sides and the included angle are known (to find a side)

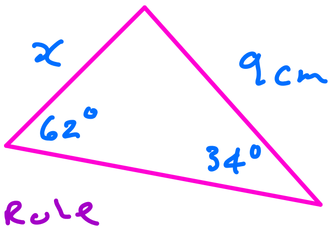
Also used to find an angle if 3 sides are known.

$$a^2 = b^2 + c^2 - 2bc \cos A$$

and $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

Exercise Find x or θ

1)

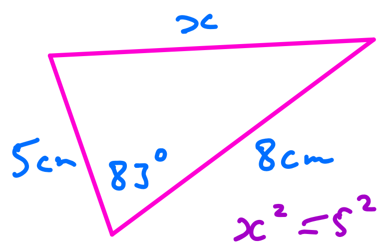


Sine Rule

$$\frac{9}{\sin 62^\circ} = \frac{x}{\sin 34^\circ}$$

$$x = \frac{9}{\sin 62^\circ} \times \sin 34^\circ = 5.7\text{cm}$$

2)



Cosine rule

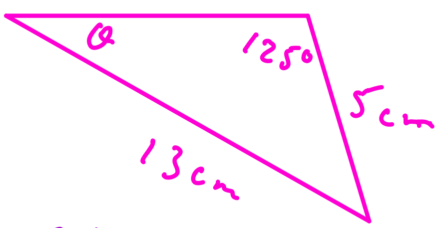
$$x^2 = 5^2 + 8^2 - 2 \times 5 \times 8 \cos 83$$

$$x^2 = 79.25$$

$$x = \sqrt{79.25}$$

$$x = 8.9\text{cm}$$

3)



Sine Rule

$$\frac{5}{\sin\theta} = \frac{13}{\sin 125}$$

$$\frac{\sin\theta}{5} = \frac{\sin 125^\circ}{13}$$

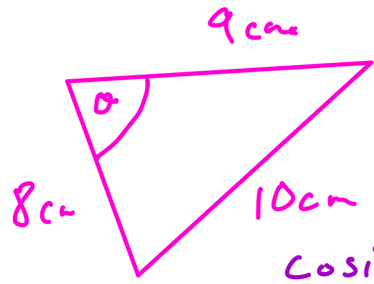
$$\sin\theta = \frac{\sin 125}{13} \times 5$$

$$\sin\theta = 0.315058$$

$$\theta = \sin^{-1}(0.315058)$$

$$\theta = 18.4^\circ$$

4)



Cosine rule

$$\cos\theta = \frac{(8^2 + 9^2 - 10^2)}{(2 \times 8 \times 9)} = \frac{5}{16}$$

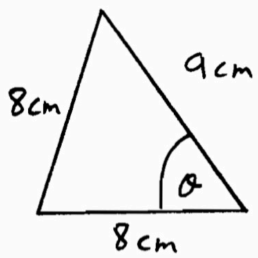
$$\theta = \cos^{-1}\left(\frac{5}{16}\right)$$

$$\theta = 71.8^\circ$$

Exercise B   Find side x or angle θ.

You must decide whether to use Sine Rule or Cosine Rule.

5)



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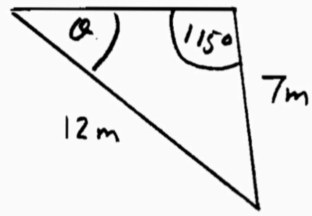

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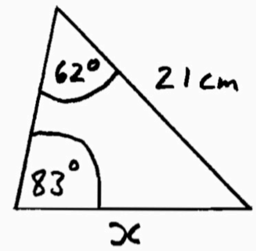

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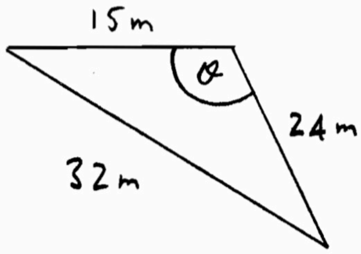

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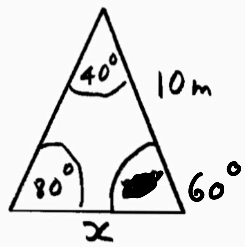

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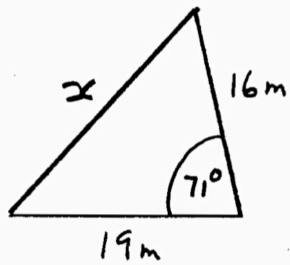
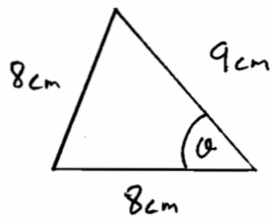


SINE AND COSINE RULES

EXERCISE

Exercise B

5.



Cosine Rule

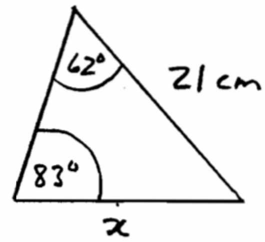
$$\cos\theta = \frac{8^2 + 9^2 - 8^2}{2 \times 8 \times 9}$$

$$\cos\theta = 0.5625$$

$$\theta = \cos^{-1} 0.5625$$

$$\theta = 55.8^\circ$$

7.



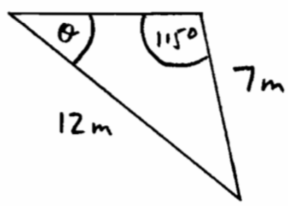
Sine Rule

$$\frac{x}{\sin 62^\circ} = \frac{21}{\sin 83^\circ}$$

$$\Rightarrow \quad x = \frac{21}{\sin 83^\circ} \times \sin 62^\circ$$

$$\Rightarrow \quad x = 18.68 \text{ cm}$$

6.



Sine Rule

$$\frac{12}{\sin 115^\circ} = \frac{7}{\sin\theta}$$

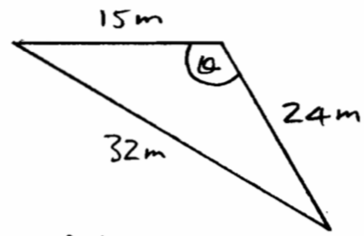
$$\Rightarrow \quad 12\sin\theta = 7\sin 115^\circ$$

$$\Rightarrow \quad \sin\theta = \frac{7\sin 115^\circ}{12}$$

$$\theta = \sin^{-1}\left(\frac{7\sin 115^\circ}{12}\right)$$

$$\theta = 31.9^\circ$$

8.



Cosine Rule

$$\cos\theta = \frac{15^2 + 24^2 - 32^2}{2 \times 15 \times 24}$$

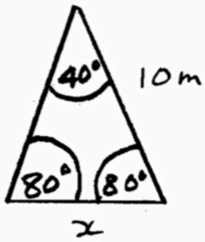
$$\cos\theta = -0.3097$$

$$\theta = 108.0^\circ$$

SINE AND COSINE RULES

EXERCISE

9.



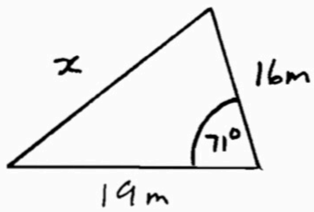
Sine Rule

$$\frac{x}{\sin 40^\circ} = \frac{10}{\sin 80^\circ}$$

$$\Rightarrow \quad x = \frac{10}{\sin 80^\circ} \times \sin 40^\circ$$

$$\Rightarrow \quad x = 6.53 \text{ m}$$

10.



Cosine Rule

$$x^2 = 19^2 + 16^2 - 2 \times 19 \times 16 \cos 71^\circ$$

$$x^2 = 419.05$$

$$x = 20.47 \text{ m}$$

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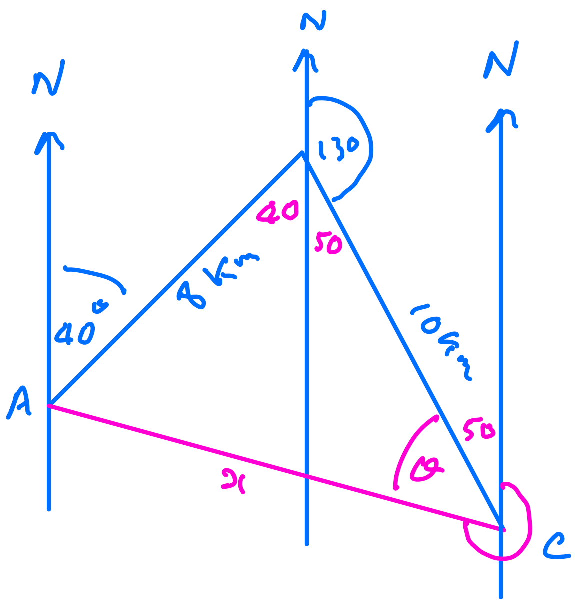
Problem Solving

A yacht leaves Port A and sails on a bearing of 040° for 8 km to B. It then sails 10 km on a bearing of 130° to C. What bearing should it sail on to go directly back to Port A and how far will that leg of the journey be?



$$x^2 = 8^2 + 10^2 - 2 \times 8 \times 10 \cos 90$$

$$x^2 = 164$$

$$x = \sqrt{164} = 12.8 \text{ km}$$

Sine rule

$$\frac{x}{\sin 90} = \frac{8}{\sin \theta}$$

$$\frac{\sin 90}{12.8} = \frac{\sin \theta}{8}$$

$$\frac{\sin 90}{12.8} \times 8 = \sin \theta$$

$$\frac{5}{8} = \sin \theta$$

$$\theta = \sin^{-1}\left(\frac{5}{8}\right)$$

$\theta = 39°$ (n'est degree)

Bearing $= 360 - (39 + 50) = 271°$