

Trigonometric Equations

From AS level

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

Exercise 5E

5 Solve the following equations for θ , in the interval $0 \leq \theta \leq 2\pi$, giving your answers to 3 significant figures where they are not exact.

a $5 \cos 2\theta = 4$

b $5 \sin 3\theta + 3 = 1$

c $\sqrt{3} \tan 4\theta - 5 = -4$

d $\sqrt{10} \cos 2\theta + \sqrt{2} = 3\sqrt{2}$

6 Solve the following equations for θ , giving your answers to 3 significant figures where appropriate, in the intervals indicated.

a $\sqrt{2} \sin 3\theta - 1 = 0, \quad -\pi \leq \theta \leq \pi$

b $2 \cos 4\theta = -1, \quad -\pi \leq \theta \leq 2\pi$

c $8 \tan 2\theta = 7, \quad -2\pi \leq \theta \leq 2\pi$

d $6 \cos 2\theta - 1 = 0.2, \quad -\pi \leq \theta \leq 3\pi$

(P) 7 Solve the following equations for θ , in the interval $0 \leq \theta \leq 2\pi$, giving your answers to 3 significant figures where they are not exact.

a $4 \cos^2 \theta = 2$

b $3 \tan^2 \theta + \tan \theta = 0$

c $\cos^2 \theta - 2 \cos \theta = 3$

d $2 \sin^2 2\theta - 5 \cos 2\theta = -2$

(P) 8 Solve the following equations for θ , in the interval $0 \leq \theta \leq 2\pi$, giving your answers to 3 significant figures where they are not exact.

a $\cos \theta + 2 \sin^2 \theta + 1 = 0$

b $10 \sin^2 \theta = 3 \cos^2 \theta$

c $4 \cos^2 \theta + 8 \sin^2 \theta = 2 \sin^2 \theta - 2 \cos^2 \theta$

d $2 \sin^2 \theta - 7 + 12 \cos \theta = 0$

(E) 9 Solve, for $0 \leq x < 2\pi$,

a $\cos\left(x - \frac{\pi}{12}\right) = \frac{1}{\sqrt{2}}$

(4 marks)

b $\sin 3x = -\frac{1}{2}$

(6 marks)

(E/P) 10 a Solve, for $-\pi \leq \theta < \pi$, $(1 + \tan \theta)(5 \sin \theta - 2) = 0$.

(4 marks)

b Solve, for $0 \leq x < 2\pi$, $4 \tan x = 5 \sin x$.

(6 marks)

7)

b $3 \tan^2 \theta + \tan \theta = 0$

d $2 \sin^2 2\theta - 5 \cos 2\theta = -2$

b)

$$3 \tan^2 \theta + \tan \theta = 0$$

$$\tan \theta (3 \tan \theta + 1) = 0$$

Either $\tan \theta = 0$ or $3 \tan \theta + 1 = 0$

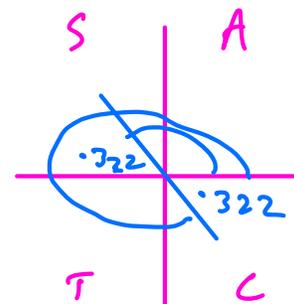
$$\theta = 0$$

$$\theta = \pi$$

$$\theta = 2\pi$$

$$3 \tan \theta = -1$$

$$\tan \theta = -\frac{1}{3}$$



$$\tan^{-1} \frac{1}{3}$$

0.322

$$\theta = \pi - 0.322$$

$$\theta = 2.810 \text{ radians}$$

$$\theta = 2\pi - 0.322$$

$$\theta = 5.961 \text{ radians}$$

7d)

$$2 \sin^2 2\theta - 5 \cos 2\theta = -2$$

$$2(1 - \cos^2 2\theta) - 5 \cos 2\theta = -2$$

$$2 - 2\cos^2 2\theta - 5\cos 2\theta = -2$$

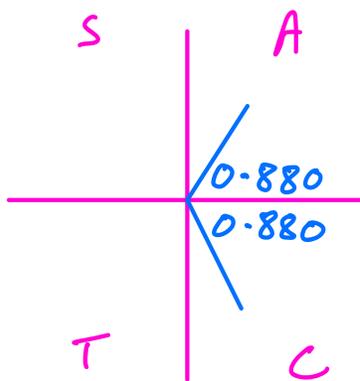
$$0 = 2\cos^2 2\theta + 5\cos 2\theta - 4$$

$$\cos 2\theta = 0.6374586088$$

$$\cos 2\theta = -3.137458609 \quad \times$$

$$\cos^{-1} 0.6374586088$$

$$= 0.880 \text{ radians}$$



$$2\theta = 0.880, \quad 2\pi - 0.880, \quad 2\pi + 0.880, \quad 4\pi - 0.880$$

$$\theta = 0.440, \quad \frac{2\pi - 0.880}{2}, \quad \frac{2\pi + 0.880}{2}, \quad \frac{4\pi - 0.880}{2}$$

$$\theta = 0.440 \quad 2.702 \quad 3.582 \quad 5.843$$

radians

$$8c) \quad 4\cos^2 \theta + 8\sin^2 \theta = 2\sin^2 \theta - 2\cos^2 \theta$$

$$6\sin^2 \theta = -6\cos^2 \theta$$

$$\frac{\sin^2 \theta}{\cos^2 \theta} = \frac{-6}{6}$$

$$\tan^2 \alpha = -1$$

No solutions

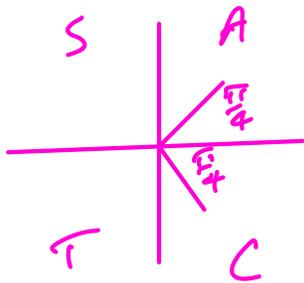
Ⓔ 9 Solve, for $0 \leq x < 2\pi$,

a $\cos\left(x - \frac{\pi}{12}\right) = \frac{1}{\sqrt{2}}$

b $\sin 3x = -\frac{1}{2}$

a) $\cos\left(x - \frac{\pi}{12}\right) = \frac{1}{\sqrt{2}}$

$$\cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$$



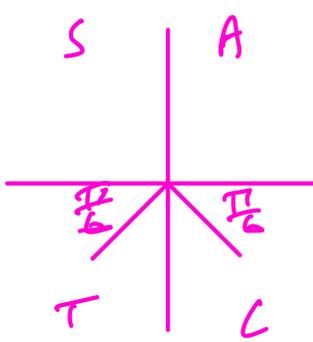
$$x - \frac{\pi}{12} = \frac{\pi}{4}, \quad \frac{7\pi}{4}$$

$$x = \frac{\pi}{4} + \frac{\pi}{12}, \quad \frac{7\pi}{4} + \frac{\pi}{12}$$

$$x = \frac{\pi}{3}, \quad x = \frac{11\pi}{6}$$

b) $\sin 3x = -\frac{1}{2}$

$$\sin^{-1}\left(-\frac{1}{2}\right) = \frac{7\pi}{6}$$



$$3x = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{19\pi}{6}, \frac{23\pi}{6}$$

$$\frac{31\pi}{6}, \frac{35\pi}{6}$$

$$x = \frac{7\pi}{18}, \frac{11\pi}{18}, \frac{19\pi}{18}, \frac{23\pi}{18}, \frac{31\pi}{18}, \frac{35\pi}{18}$$
