

Trinomial Expansions

$$\begin{aligned}\text{Ex1} \quad & (2x + 1)(x + 2)(3x + 4) \\ &= [2x^2 + x + 4x + 2](3x + 4) \\ &= [2x^2 + 5x + 2](3x + 4) \\ &= \begin{array}{r} 6x^3 + 15x^2 + 6x \\ + 8x^2 + 20x + 8 \end{array} \\ &= \hline &= 6x^3 + 23x^2 + 26x + 8 \end{aligned}$$

$$\begin{aligned}\text{Ex2} \quad & (2x - 3)(3x + 1)(4x - 5) \\ &= [6x^2 - 9x + 2x - 3](4x - 5) \\ &= [6x^2 - 7x - 3](4x - 5) \\ &= \begin{array}{r} 24x^3 - 28x^2 - 12x \\ - 30x^2 + 35x + 15 \end{array} \\ &= \hline &= 24x^3 - 58x^2 + 23x + 15 \end{aligned}$$

Exercise

$$\begin{aligned}
1) \quad & (x+3)(2x+3)(x+5) \\
& = [2x^2 + 6x + 3x + 9](x+5) \\
& = [2x^2 + 9x + 9](x+5) \\
& = \begin{array}{r} 2x^3 + 9x^2 + 9x \\ + 10x^2 + 45x + 45 \end{array} \\
& = \underline{2x^3 + 19x^2 + 54x + 45}
\end{aligned}$$

$$\begin{aligned}
2) \quad & (2x-3)(2x+5)(x+1) \\
& = [4x^2 - 6x + 10x - 15](x+1) \\
& = [4x^2 + 4x - 15](x+1) \\
& = \begin{array}{r} 4x^3 + 4x^2 - 15x \\ + 4x^2 + 4x - 15 \end{array} \\
& = \underline{4x^3 + 8x^2 - 11x - 15}
\end{aligned}$$

$$\begin{aligned}
3) \quad & (2x-1)(3x-2)(4x-3) \\
& = [6x^2 - 3x - 4x + 2](4x-3) \\
& = [6x^2 - 7x + 2](4x-3) \\
& = \begin{array}{r} 24x^3 - 28x^2 + 8x \\ - 18x^2 + 21x - 6 \end{array} \\
& = \underline{24x^3 - 46x^2 + 29x - 6}
\end{aligned}$$

$$= 24x^3 - 46x^2 + 29x - 6$$

Completing the Square

$$\begin{aligned}(x+a)^2 &= (x+a)(x+a) \\ &= x^2 + 2ax + a^2\end{aligned}$$

Ex1

$$\begin{aligned}x^2 + 6x + 5 \\ &= (x+3)^2 + 5 - 9 \\ &= (x+3)^2 - 4\end{aligned}$$

Ex2

$$\begin{aligned}x^2 - 8x + 20 \\ &= (x-4)^2 + 20 - 16 \\ &= (x-4)^2 + 4\end{aligned}$$

Ex3

$$\begin{aligned}x^2 + 5x + 10 \\ &= \left(x + \frac{5}{2}\right)^2 + 10 - \frac{25}{4} \\ &= \left(x + \frac{5}{2}\right)^2 + \frac{40}{4} - \frac{25}{4} \\ &= \left(x + \frac{5}{2}\right)^2 + \frac{15}{4}\end{aligned}$$

Exercise Complete the Squares

1)

$$\begin{aligned}x^2 + 10x + 3 \\ &= (x+5)^2 + 3 - 25\end{aligned}$$

$$= (x+5)^2 - 22$$

$$\begin{aligned} 2) \quad & x^2 - 9x - 1 \\ &= \left(x - \frac{9}{2}\right)^2 - 1 - \frac{81}{4} \\ &= \left(x - \frac{9}{2}\right)^2 - \frac{44}{4} - \frac{81}{4} \\ &= \left(x - \frac{9}{2}\right)^2 - \frac{85}{4} \end{aligned}$$

$$\begin{aligned} 3) \quad & x^2 + x + 7 \\ &= \left(x + \frac{1}{2}\right)^2 + 7 - \frac{1}{4} \\ &= \left(x + \frac{1}{2}\right)^2 + \frac{28}{4} - \frac{1}{4} \\ &= \left(x + \frac{1}{2}\right)^2 + \frac{27}{4} \end{aligned}$$

$$\begin{aligned} 4) \quad & x^2 - 2x + 3 \\ &= (x-1)^2 + 3 - 1 \\ &= (x-1)^2 + 2 \end{aligned}$$

Translations of Graphs

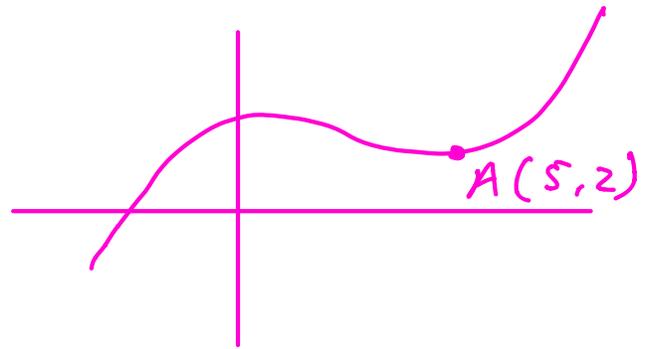
if $y = f(x)$

$y = f(x-a)$ is a translation
by $\begin{pmatrix} a \\ 0 \end{pmatrix}$

$y = f(x) + b$ is a translation
by $\begin{pmatrix} 0 \\ b \end{pmatrix}$

$y = f(x - a) + b$ is a translation
by $\begin{pmatrix} a \\ b \end{pmatrix}$

Let $y = f(x)$



What is the image of A on the graph

1) $y = f(x - 3) + 7$

$$\begin{aligned} \text{Image of } A &= (5 + 3, 2 + 7) \\ &= (8, 9) \end{aligned}$$

2) $y = f(x + 1) + 3$

$$\begin{aligned} \text{Image of } A &= (5 - 1, 2 + 3) \\ &= (4, 5) \end{aligned}$$

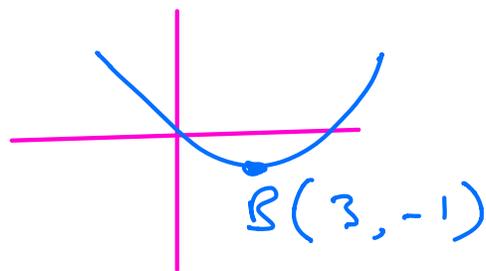
3) $y = f(x + \frac{1}{2}) - \frac{1}{2}$

$$\text{Image } A = (5 - \frac{1}{2}, 2 - \frac{1}{2})$$

$$= \left(\frac{a}{2}, \frac{3}{2} \right)$$

Exercise

$$y = f(x)$$



Find image of B

$$y = f(x+2) + 3$$

$$\begin{aligned} \text{Image of B} &= (3-2, -1+3) \\ &= (1, 2) \end{aligned}$$

$$y = f(x) = x^2$$

$$y = f(x-3) + 4$$

What happens to C the origin

$$(0, 0) \rightarrow (3, 4)$$