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Candidate surname		Other names	
Pearson Edexcel		Centre Number	Candidate Number
Level 3 GCE		<input type="text"/>	<input type="text"/>
Sample Assessment Material			
(Time: 1 hour 30 minutes)		Paper Reference 9FM0/01	
Further Mathematics Advanced Paper 1: Core Pure Mathematics 1			
You must have: Mathematical Formulae and Statistical Tables, calculator			Total Marks

Solutions

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear.
Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

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Pearson

Answer ALL questions. Write your answers in the spaces provided.

1. Solve the equation

$$6 \cosh 2x + 4 \sinh x = 7$$

giving your answers as exact logarithms.

$$6(1 + 2 \sinh^2 x) + 4 \sinh x = 7 \quad (6)$$

$$12 \sinh^2 x + 4 \sinh x - 1 = 0$$

$$(6 \sinh x - 1)(2 \sinh x + 1) = 0$$

$$\sinh x = \frac{1}{6} \quad \text{or} \quad \sinh x = -\frac{1}{2}$$

$$x = \ln\left(\frac{1}{6} + \sqrt{\left(\frac{1}{6}\right)^2 + 1}\right)$$

$$x = \ln\left(-\frac{1}{2} + \sqrt{\frac{1}{4} + 1}\right)$$

$$x = \ln\left(\frac{1 + \sqrt{37}}{6}\right)$$

$$x = \ln\left(\frac{-1 + \sqrt{5}}{2}\right)$$



2. A company runs three theme parks, *A* (Aztec Adventureland), *B* (Babylonian Towers) and *C* (Carthaginian Kingdom).

It is known that park *A* makes a profit of £30 per visitor, park *B* makes a profit of £26 per visitor and park *C* makes a profit of £33 per visitor.

In 2017 the Aztec Adventureland park was upgraded, which took one year to carry out. During 2017

- park *A* had only 50% of the number of visitors it had in 2016
- park *B* had 25% more than the number of visitors it had in 2016
- park *C* had 15% more than the number of visitors it had in 2016

In total 1 350 000 people visited the three theme parks during 2017.

The company made a total profit from the parks of £39.15 million in 2016. The profits dropped by 1% for 2017. $£39.15m \times 0.99 = £38.7585m$

Form and solve a matrix equation to find, to 2 significant figures, the number of visitors for each of the theme parks in 2016.

Let numbers for *A*, *B*, *C* be x , y , z in 2016 (8)

$$30x + 26y + 33z = 39,150,000$$

$$0.5(30x) + 1.25(26y) + 1.15(33z) = 38,758,500$$

$$0.5x + 1.25y + 1.15z = 1,350,000$$

$$\begin{pmatrix} 30 & 26 & 33 \\ 15 & 32.5 & 37.95 \\ 0.5 & 1.25 & 1.15 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 39,150,000 \\ 38,758,500 \\ 1,350,000 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \underline{\underline{M}}^{-1} \begin{pmatrix} 39,150,000 \\ 38,758,500 \\ 1,350,000 \end{pmatrix} = \begin{pmatrix} 400,332 \\ 593,257 \\ 355,011 \end{pmatrix}$$

Visitors in 2016 to 2 s.f

$$A = 400,000 \quad B = 590,000 \quad C = 360,000$$



3.

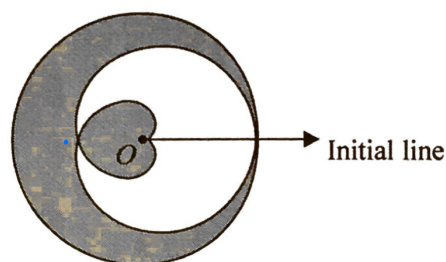


Figure 1

Figure 1 shows a sketch for the design of a logo. The logo is defined by the polar curve with equation

$$r = \sin\left(\frac{\theta}{6}\right) \quad 0 \leq \theta \leq 6\pi$$

The inner closed section and outer closed section of the curve, shown shaded in Figure 1, are to be coloured the same colour. The remaining section is to be left clear.

(a) Use algebraic integration to find the area of the coloured sections of the logo.

(6)

A copy of this logo is to be painted on a white wall of a building such that the total width of the logo is 12 m.

Tins of coloured paint with an advertised minimum coverage area of 30 m² are to be used to paint the coloured sections of the logo onto the wall. Given that two coats of paint will be needed,

(b) find the minimum number of tins of this paint that should be bought to ensure that the coloured sections of the logo can be painted onto the wall.

(4)

a)

θ	0	π	2π	3π	4π	5π	6π
$\sin \frac{\theta}{6}$	0	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	0

$$\text{Required Area} = 2 \left[\int_0^{\pi} \frac{1}{2} r^2 d\theta + \int_{2\pi}^{3\pi} \frac{1}{2} r^2 d\theta - \int_{\pi}^{2\pi} \frac{1}{2} r^2 d\theta \right]$$

$$\int_a^b \frac{1}{2} r^2 d\theta = \int_a^b \frac{1}{2} \sin^2 \frac{\theta}{6} d\theta = \frac{1}{2} \int_a^b \left(\frac{1 - \cos \frac{\theta}{3}}{2} \right) d\theta$$



Question 3 continued

$$= \frac{1}{4} \left[\theta - 3 \sin \frac{\theta}{3} \right]_a^b$$

$$\begin{aligned} \text{Area} &= 2 \left[\frac{1}{4} \left[\pi - 3 \sin \frac{\pi}{3} \right] + \frac{1}{4} \left[3\pi - 3 \sin \pi - (2\pi - 3 \sin \frac{2\pi}{3}) \right] \right. \\ &\quad \left. - \frac{1}{4} \left[2\pi - 3 \sin \frac{2\pi}{3} - (\pi - 3 \sin \frac{\pi}{3}) \right] \right] \\ &= \frac{1}{2} \left[\pi - \frac{3\sqrt{3}}{2} + 3\pi - 0 - 2\pi + \frac{3\sqrt{3}}{2} - 2\pi + \frac{3\sqrt{3}}{2} + \pi - \frac{3\sqrt{3}}{2} \right] \\ &= \frac{1}{2} \left[\pi \right] = \frac{\pi}{2} \end{aligned}$$

b) Width $= 1 + \frac{\sqrt{3}}{2} = \frac{2+\sqrt{3}}{2}$ gives Area $\frac{\pi}{2}$

Linear scale $\frac{2+\sqrt{3}}{2} : 12 = 2+\sqrt{3} : 24$

Area scale $(2+\sqrt{3})^2 : 24^2$

Painted area $\therefore \frac{\pi}{2} \times \frac{24^2}{(2+\sqrt{3})^2}$

$$= 64.96 \text{ m}^2$$

2 coats so 129.92 m^2

Each tin covers 30 m^2 so 5 tins required



S 6 1 2 9 4 A 0 9 2 8

4.

$$f(x) = \begin{cases} \frac{kx}{x^2 + 6} & \text{for } 0 \leq x < 3 \\ \frac{k}{x^2 - 4} & \text{for } 3 \leq x \end{cases}$$

where k is a positive constant.

The area between the curve $y = f(x)$ and the positive x -axis is $\frac{1}{4}$

Show that

$$k = \frac{1}{\ln a}$$

where a is a constant to be determined.

$$\text{Area} = \int_0^3 \frac{kx}{x^2 + 6} dx + \int_3^\infty \frac{k}{x^2 - 4} dx \quad (8)$$

$$\left[\frac{1}{x^2 - 4} = \frac{1}{(x+2)(x-2)} = -\frac{\frac{1}{4}}{(x+2)} + \frac{\frac{1}{4}}{(x-2)} \right]$$

$$= \frac{k}{2} \int_0^3 \frac{2x}{x^2 + 6} dx + \frac{k}{4} \int_3^\infty \left(\frac{1}{x-2} - \frac{1}{x+2} \right) dx$$

$$= \frac{k}{2} \left[\ln(x^2 + 6) \right]_0^3 + \frac{k}{4} \left[\ln(x-2) - \ln(x+2) \right]_3^\infty$$

$$= \frac{k}{2} \left[\ln 15 - \ln 6 \right] + \frac{k}{4} \left[\ln \left(\frac{x-2}{x+2} \right) \right]_3^\infty$$

$$= \frac{k}{2} \ln \left(\frac{15}{6} \right) + \frac{k}{4} \left[\ln 1 - \ln \left(\frac{1}{5} \right) \right]$$



Question 4 continued

$$= \frac{k}{2} \ln\left(\frac{5}{2}\right) + \frac{k}{4} \ln 5$$

$$\text{Area} = \frac{1}{4}$$

$$\therefore \frac{k}{2} \ln\left(\frac{5}{2}\right) + \frac{k}{4} \ln 5 = \frac{1}{4}$$

$$2k \ln\left(\frac{5}{2}\right) + k \ln 5 = 1$$

$$k \ln\left(\frac{25}{4}\right) + k \ln 5 = 1$$

$$k \left(\ln\left(\frac{25}{4}\right) + \ln 5 \right) = 1$$

$$k \ln\left(\frac{125}{4}\right) = 1$$

$$k = \frac{1}{\ln\left(\frac{125}{4}\right)}$$

(Total for Question 4 is 8 marks)



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5.

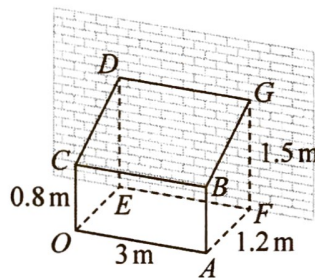


Figure 2

Figure 2 shows a sketch of a shelter against a wall. The shelter consists of two rectangular wooden boards, $OABC$ and $BCDG$, which can be modelled as parts of planes. Board $OABC$ is vertical and parallel to the wall and the ground may be assumed to be horizontal.

The points E and F are at the foot of the wall directly below D and G respectively.

The length OC is 0.8 m , the length OA is 3 m and the board $OABC$ is 1.2 m away from the wall. The points D and G are 1.5 m above the ground.

To model the shelter, take O as the origin, the vector \mathbf{i} to be 1 m in the direction of \vec{OA} , the vector \mathbf{j} to be 1 m in the direction of \vec{OE} and the vector \mathbf{k} to be 1 m in the direction of \vec{OC} .

- (a) Find an equation of the plane $BCDG$, giving your answer in the form $\mathbf{r} \cdot \mathbf{n} = d$ (5)

In order to support the roof of the shelter, one end of a pole is attached to the ground at the centre of the rectangle $OAFE$ and the other end to a point on the roof. Modelling the pole as a rod,

- (b) find, to the nearest mm, the shortest possible length for the pole. (3)

- (c) State a limitation of the assumption that the boards can be modelled as planes. (1)

$$\vec{BG} = \vec{BA} + \vec{AF} + \vec{FG}$$

$$= -0.8\mathbf{k} + 1.2\mathbf{j} + 1.5\mathbf{k}$$

$$\vec{BG} = \begin{pmatrix} 0 \\ 1.2 \\ 0.7 \end{pmatrix}$$

Normal $\begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} \perp$ to \vec{BG} and \vec{CB}



Question 5 continued

$$\begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix} = 0 \Rightarrow n_1 = 0$$

$$\begin{pmatrix} 0 \\ n_2 \\ n_3 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1.2 \\ 0.7 \end{pmatrix} = 0$$

$$\Rightarrow 1.2 n_2 + 0.7 n_3 = 0$$

$$n_2 = \frac{-0.7 n_3}{1.2}$$

$$\text{Let } n_3 = 12$$

$$\Rightarrow n_2 = -7$$

$$\vec{n} = \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -7 \\ 12 \end{pmatrix}$$

$$\text{Plane } \vec{r} \cdot \begin{pmatrix} 0 \\ -7 \\ 12 \end{pmatrix} = d$$

$C(0,0,0.8)$ on plane

$$\begin{pmatrix} 0 \\ 0 \\ 0.8 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ -7 \\ 12 \end{pmatrix} = d$$

$$d = 9.6$$



S 6 1 2 9 4 A 0 1 5 2 8

Question 5 continued

Plane is $\vec{r} \cdot \begin{pmatrix} 0 \\ -7 \\ 12 \end{pmatrix} = 9.6$

b) Plane is $0x - 7y + 12z - 9.6 = 0$

Point on ground $(1.5, 0.6, 0)$

\perp distance to plane

$$= \frac{|0(1.5) + (-7)(0.6) + 12(0) - 9.6|}{\sqrt{0^2 + (-7)^2 + 12^2}}$$

$$= 0.9933 \text{ m}$$

$$= 99 \text{ cm } 3 \text{ mm to nearest mm}$$

c) Boards could warp



6.

$$f(x) = kx^2 + 3x - 11$$

$$g(x) = mx^3 - 2x^2 + 3x - 9$$

where k and m are real constants.

Given that

- the sum of the roots of f is equal to the product of the roots of g
- g has at least one root on the imaginary axis

(a) solve completely

(i) $f(x) = 0$

(ii) $g(x) = 0$

(7)

(b) Plot the roots of f and the roots of g on a single Argand diagram.

(2)

a)

$$-\frac{3}{k} = \frac{9}{m}$$

Roots of $g(x) = 0$ hi and $-hi$ for some real h

$$\text{Third root } \gamma = \frac{9}{m} = \frac{9}{mh^2}$$

$$\alpha = hi \quad \beta = -hi \quad \gamma = \frac{9}{mh^2}$$

$$\alpha + \beta + \gamma = \frac{9}{mh^2} = \frac{2}{m}$$

$$= 9m = 2mh^2$$

$$\frac{9}{2} = h^2$$

$$\frac{3}{\sqrt{2}} = h$$



Question 6 continued

$$\alpha\beta + \alpha\gamma + \beta\gamma = +\frac{3}{m}$$

$$= h^2 + (hi - hi)\gamma = +\frac{3}{m}$$

$$= h^2 = \frac{3}{m}$$

$$\frac{9}{2} = \frac{3}{m}$$

$$9m = 6$$

$$m = \frac{6}{9} = \frac{2}{3}$$

Roots

of $q(x)=0$

$$\alpha = hi = \frac{3}{\sqrt{2}}i$$

$$\beta = -hi = -\frac{3}{\sqrt{2}}i$$

$$\gamma = \frac{9}{mh^2} = \frac{9}{\frac{2}{3} \times \frac{9}{2}} = 3$$

$$\text{Now } -\frac{3}{k} = \frac{9}{m} = \frac{9}{\frac{2}{3}} = \frac{27}{2}$$

$$\Rightarrow 27k = -6$$

$$k = -\frac{2}{9}$$

(Total for Question 6 is 9 marks)



S 6 1 2 9 4 A 0 1 9 2 8

$$f(x)=0 \quad -\frac{2}{9}x^2 + 3x - 11 = 0$$

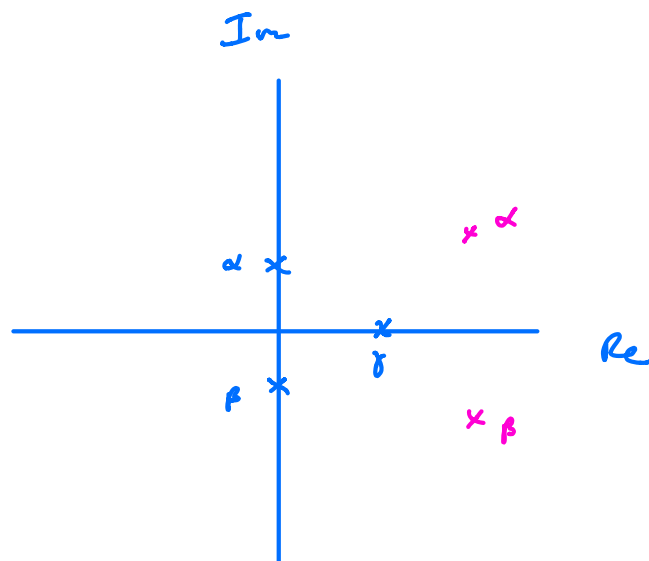
$$2x^2 - 27x + 99 = 0$$

$$\text{By calc} \quad x = \frac{27 \pm 3\sqrt{7}i}{4}$$

Roots of $f(x) = 0$

$$\alpha = \frac{27 + 3\sqrt{7}i}{4} \quad \beta = \frac{27 - 3\sqrt{7}i}{4}$$

b)



7. (i) Prove by induction that, for $n \in \mathbb{N}$,

$$\begin{pmatrix} 4 & -1 \\ 9 & -2 \end{pmatrix}^n = \begin{pmatrix} 3n+1 & -n \\ 9n & 1-3n \end{pmatrix}$$

(6)

- (ii) Consider the statement

$$n^2 < 2^n \quad \text{for all } n \in \mathbb{Z}^+$$

A student attempts to prove this statement using induction as follows.

Student's response

For $n = 1$ we have $1^2 = 1$ and $2^1 = 2$

Since $1 < 2$ the statement is true for $n = 1$

Suppose it is true for $n = k$, so $k^2 < 2^k$

Line 4 \rightarrow

$$\begin{aligned} \text{Then } (k+1)^2 &= k^2 + 2k + 1 < k^2 + k^2 && (\text{since } 2k + 1 < k^2 \text{ for } k \in \mathbb{Z}^+) \\ &= 2k^2 \\ &< 2 \times 2^k && (\text{by the assumption } k^2 < 2^k) \\ &= 2^{k+1} \end{aligned}$$

Hence the result is true for $n = k + 1$

So the result is true for $n = 1$ and if it is true for $n = k$ then it is true for $n = k + 1$, and hence it is true for all positive integers n by mathematical induction.

- (a) Show by a counterexample that the statement is not true.

Given that the only mathematical error in the student's proof occurs in line 4,

- (b) identify the error made in the student's proof,

- (c) hence determine for which positive integers the statement is true, explaining your reasoning.

(5)

- (i) Prove by induction that, for $n \in \mathbb{N}$,

$$\begin{pmatrix} 4 & -1 \\ 9 & -2 \end{pmatrix}^n = \begin{pmatrix} 3n+1 & -n \\ 9n & 1-3n \end{pmatrix}$$

$$n=1 \quad \begin{pmatrix} 4 & -1 \\ 9 & -2 \end{pmatrix}^1 = \begin{pmatrix} 3+1 & -1 \\ 9 & 1-3 \end{pmatrix} \quad \checkmark \quad \text{true for } n=1$$

$$\text{Assume true for } n=k \quad \begin{pmatrix} 4 & -1 \\ 9 & -2 \end{pmatrix}^k = \begin{pmatrix} 3k+1 & -k \\ 9k & 1-3k \end{pmatrix}$$



Question 7 continued

Consider $\begin{pmatrix} 4 & -1 \\ 9 & -2 \end{pmatrix}^{k+1}$

$$= \begin{pmatrix} 3k+1 & -k \\ 9k & 1-3k \end{pmatrix} \begin{pmatrix} 4 & -1 \\ 9 & -2 \end{pmatrix} = \begin{pmatrix} 4(3k+1) - 9k & -3k-1+2k \\ 36k+9(1-3k) & -9k-2(1-3k) \end{pmatrix}$$

$$= \begin{pmatrix} 3k+4 & -k-1 \\ 9k+9 & -3k-2 \end{pmatrix}$$

$$= \begin{pmatrix} 3(k+1)+1 & -(k+1) \\ 9(k+1) & 1-3(k+1) \end{pmatrix}$$

Same matrix with k replaced by $k+1$

\therefore if true for $n=k$ also true for $n=k+1$

Since true for $n=1$, by mathematical induction
it is true for all $n \in \mathbb{N}$

ii)

a) $3^2 = 9$ $2^3 = 8$ $\therefore 3^2 < 2^3$ is false

b) $2k+1 < k^2$ is false when $k=2$

$$2(2)+1 \neq 2^2$$

$$5 > 4$$

c) $2k+1 < k^2$ for $k \geq 3$

$$k^2 < 2^k \text{ for } k=5$$

so true for $n \geq 5$

$k^2 < 2^k$ not true for $n=2,3,4$

and $n=1$



S 6 1 2 9 4 A 0 2 1 2 8

8. A large container initially contains 3 litres of pure water. Contaminated water starts pouring into the container at a constant rate of 250 ml per minute and you may assume the contaminant dissolves completely.

At the same time, the container is drained at a constant rate of 125 ml per minute. The water in the container is continually mixed.

The amount of contaminant in the water pouring into the container, at time t minutes after pouring began, is modelled to be $(5 - e^{-0.1t})$ mg per litre.

Let m be the amount of contaminant, in milligrams, in the container at time t minutes after the contaminated water begins pouring into the container.

- (a) (i) Write down an expression for the total volume of water in litres in the container at time t .
- (ii) Hence show that the amount of contaminant in the container can be modelled by the differential equation

$$\frac{dm}{dt} = \frac{5 - e^{-0.1t}}{4} - \frac{m}{24 + t} \quad (4)$$

- (b) By solving the differential equation, find an expression for the amount of contaminant, in milligrams, in the container t minutes after the contaminated water begins to be poured into the container.

(8)

After 30 minutes, the concentration of contaminant in the water was measured as 3.79 mg per litre.

- (c) Assess the model in light of this information, giving a reason for your answer.

(2)

a) i)

$$V = 3 + 0.25t - 0.125t$$

$$V = 3 + 0.125t = 3 + \frac{t}{8}$$

ii) At time t amount of contaminant = m

$$\begin{aligned} \text{Rate it is leaving} &= m \times \frac{0.125}{3 + \frac{t}{8}} = \frac{m \times \frac{1}{8}}{\frac{24+t}{8}} \\ &= \frac{m}{24+t} \end{aligned}$$



Question 8 continued

$$\text{Rate it is entering} = 0.25(5 - e^{-0.1t})$$

$$\text{Overall } \frac{dm}{dt} = 0.25(5 - e^{-0.1t}) - \frac{m}{24+t}$$

$$\frac{dm}{dt} = \frac{5 - e^{-0.1t}}{4} - \frac{m}{24+t}$$

$$b) \quad IF = e^{\int \frac{1}{24+t} dt} = e^{\ln(24+t)} = 24+t$$

$$(24+t) \frac{dm}{dt} + m = \frac{5 - e^{-0.1t}}{4} (24+t)$$

$$\frac{d}{dt} (24+t)m = \frac{1}{4} [120 - 24e^{-0.1t} + 5t - te^{-0.1t}]$$

$$4(24+t)m = \int (120 - 24e^{-0.1t} + 5t - te^{-0.1t}) dt$$

$$\left[\int -te^{-0.1t} dt \quad \begin{array}{l} \text{Let } u = -t \\ \frac{du}{dt} = -1 \end{array} \quad \begin{array}{l} \frac{dv}{dt} = e^{-0.1t} \\ v = -10e^{-0.1t} \end{array} \right]$$

$$\int u \frac{dv}{dt} = uv - \int v \frac{du}{dt}$$

$$\begin{aligned} \int -te^{-0.1t} dt &= 10te^{-0.1t} - \int 10e^{-0.1t} dt \\ &= 10te^{-0.1t} + 100e^{-0.1t} \end{aligned}$$



S 6 1 2 9 4 A 0 2 5 2 8

Question 8 continued

$$4(24+t)m = 120t + 240e^{-0.1t} + \frac{5t^2}{2} + 10te^{-0.1t} + 100e^{-0.1t}$$

$$4(24+t)m = 120t + \frac{5t^2}{2} + (340 + 10t)e^{-0.1t} + c$$

$$m = \frac{240t + 5t^2 + (680 + 20t)e^{-0.1t} + c}{8(24+t)}$$

$$m=0, t=0$$

$$0 = \frac{680}{192} + \frac{c}{192}$$

$$-680 = c$$

$$m = \frac{240t + 5t^2 + (680 + 20t)e^{-0.1t} - 680}{8(24+t)}$$

$$c) \quad t=30, \quad V = 3 + \frac{30}{8} = 6.75 \text{ litres}$$

$$\text{Contaminant} = 6.75 \times 3.79 = 25.58 \text{ mg}$$

Test Model

$$m = \frac{240(30) + 5(30)^2 + (680 + 20(30))e^{-3} - 680}{8 \times 54}$$

$$= 25.66 \text{ mg}$$

Model appears to be very accurate

Predicted value very close to measured value

