Direct Proportion

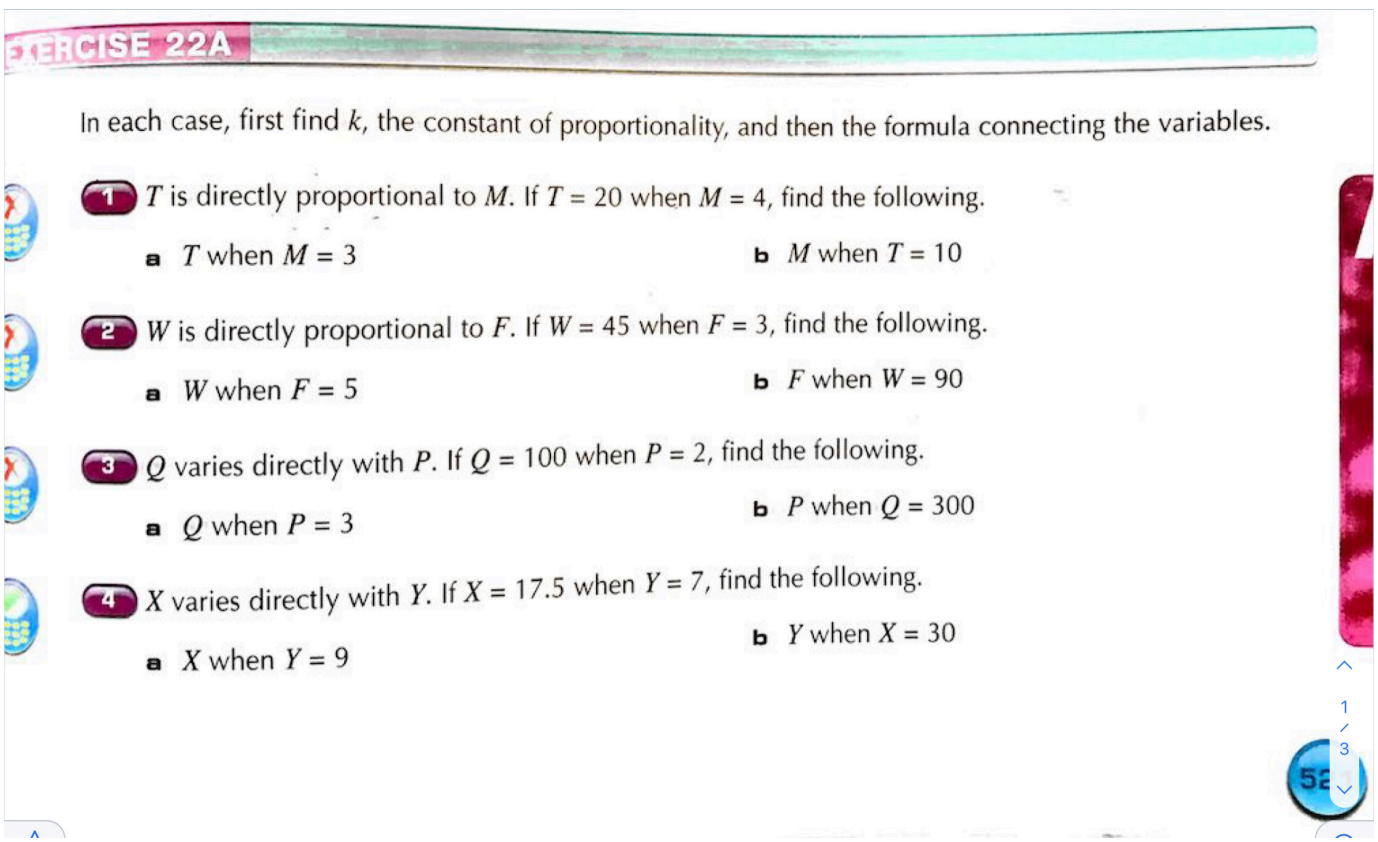
A variable y is directly proportional to a variable x if it is a constant multiple of x

We write $y = kx$

where k is the constant of proportionality

Examples

$y = 2x$ $\quad y = \frac{1}{3}x$ $\quad y = 0.586x$



EXERCISE 22A

In each case, first find k, the constant of proportionality, and then the formula connecting the variables.

1. T is directly proportional to M. If $T = 20$ when $M = 4$, find the following.
   a T when $M = 3$ b M when $T = 10$

2. W is directly proportional to F. If $W = 45$ when $F = 3$, find the following.
   a W when $F = 5$ b F when $W = 90$

3. Q varies directly with P. If $Q = 100$ when $P = 2$, find the following.
   a Q when $P = 3$ b P when $Q = 300$

4. X varies directly with Y. If $X = 17.5$ when $Y = 7$, find the following.
   a X when $Y = 9$ b Y when $X = 30$

1) $T \propto M$ $\quad T = kM$

$T = 20$ when $M = 4$

$20 = 4k$

$\frac{20}{4} = k$

$5 = k$

$T = 5M$

a) Find T when $M = 3$

$T = 5 \times 3$

$T = 15$

b) Find M when $T = 10$

$10 = 5M$

$\frac{10}{5} = M$

$M = 2$

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3) $Q = kP$ $\quad \begin{cases} Q = 100 \\ P = 2 \end{cases}$

$100 = 2k$

$\frac{100}{2} = k$

$50 = k$

$Q = 50P$

a) Find Q when $P = 3$

$Q = 50 \times 3$

$Q = 150$

b) Find P when $Q = 300$

$300 = 50P$

$\frac{300}{50} = P$

$P = 6$

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2) $W = kF$

$45 = 3k$

$\frac{45}{3} = k$

$15 = k$

$\begin{cases} W = 45 \\ F = 3 \end{cases}$

$\underline{W = 15F}$

a) Find W when F=5

$W = 15 \times 5$

$\underline{W = 75}$

b) Find F when W = 90

$90 = 15F$

$\frac{90}{15} = F$

$\underline{F = 6}$

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4) $X = kY$

$17.5 = 7k$

$\frac{17.5}{7} = k$

$2.5 = k$

$\begin{cases} x = 17.5 \\ Y = 7 \end{cases}$

$X = 2.5Y$ or $\frac{5Y}{2}$

a) Find x when Y=9

$X = 2.5 \times 9$

$\underline{X = 22.5}$

b) Find Y when X = 30

$30 = 2.5Y$

$\frac{30}{2.5} = Y$

$\underline{Y = 12}$

EXERCISE 22B

In each case, first find k, the constant of proportionality, and then the formula connecting the variables.

1 T is directly proportional to x^2. If $T = 36$ when $x = 3$, find the following.

a T when $x = 5$ **b** x when $T = 400$

2 W is directly proportional to M^2. If $W = 12$ when $M = 2$, find the following.

a W when $M = 3$ **b** M when $W = 75$

3 E varies directly with $\sqrt{C}$. If $E = 40$ when $C = 25$, find the following.

a E when $C = 49$ **b** C when $E = 10.4$

4 X is directly proportional to $\sqrt{Y}$. If $X = 128$ when $Y = 16$, find the following.

a X when $Y = 36$ **b** Y when $X = 48$

5 P is directly proportional to f^3. If $P = 400$ when $f = 10$, find the following.

a P when $f = 4$ **b** f when $P = 50$

6 The cost of serving tea and biscuits varies directly with the square root of the number of people at the buffet. It costs £25 to serve tea and biscuits to 100 people.

a How much will it cost to serve tea and biscuits to 400 people?

b For a cost of £37.50, how many could be served tea and biscuits?

7 In an experiment, the temperature, in °C, varied directly with the square of the pressure, in atmospheres. The temperature was 20 °C when the pressure was 5 atm.

a What will the temperature be at 2 atm? **b** What will the pressure be at 80 °C?

8 The weight, in grams, of ball bearings varies directly with the cube of the radius measured in millimetres. A ball bearing of radius 4 mm has a weight of 115.2 g.

a What will a ball bearing of radius 6 mm weigh?

b A ball bearing has a weight of 48.6 g. What is its radius?

9 The energy, in J, of a particle varies directly with the square of its speed in m/s. A particle moving at 20 m/s has 50 J of energy.

a How much energy has a particle moving at 4 m/s?

b At what speed is a particle moving if it has 200 J of energy?

10 The cost, in £, of a trip varies directly with the square root of the number of miles travelled. The cost of a 100-mile trip is £35.

a What is the cost of a 500-mile trip (to the nearest £1)?

b What is the distance of a trip costing £70?

2 W is directly proportional to M^2. If $W = 12$ when $M = 2$, find the following.

a W when $M = 3$ **b** M when $W = 75$

$W = kM^2$

$12 = k \times 2^2$

$12 = 4k$

$\frac{12}{4} = k$

$3 = k$

$\begin{cases} W = 12 \\ M = 2 \end{cases}$

$W = 3M^2$

a) Find W when $M = 3$

$W = 3 \times 3^2$

$W = 27$

b) Find M when $W = 75$

$75 = 3M^2$

$\frac{75}{3} = M^2$

$25 = M^2$

$\sqrt{25} = M$

$M = \pm 5$

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4)

4 X is directly proportional to $\sqrt{Y}$. If $X = 128$ when $Y = 16$, find the following.

a X when $Y = 36$ **b** Y when $X = 48$

$X = k\sqrt{Y}$

$128 = k \times \sqrt{16}$

$128 = 4k$

$\frac{128}{4} = k$

$32 = k$

$\begin{cases} X = 128 \\ Y = 16 \end{cases}$

$X = 32\sqrt{Y}$

a) Find x when $y = 36$

$$x = 32\sqrt{36}$$
$$x = 32 \times 6$$
$$x = 192$$

b) Find y when $x = 48$

$$48 = 32\sqrt{y}$$
$$\frac{48}{32} = \sqrt{y}$$
$$\frac{3}{2} = \sqrt{y}$$
$$\left(\frac{3}{2}\right)^2 = y$$
$$y = \frac{9}{4}$$

CLASS WORK

1 T is directly proportional to x^2. If $T = 36$ when $x = 3$, find the following.

a T when $x = 5$ **b** x when $T = 400$

$T = kx^2$

$\begin{cases} T = 36 \\ x = 3 \end{cases}$

$36 = k \times 3^2$

$36 = 9k$

$\frac{36}{9} = k$

$4 = k$

$T = 4x^2$

a) Find T when $x = 5$

$T = 4 \times 5^2$

$T = 4 \times 25$

$T = 100$

b) Find x when $T = 400$

$400 = 4x^2$

$\frac{400}{4} = x^2$

$100 = x^2$

$\sqrt{100} = x$

$x = \pm 10$

3 E varies directly with $\sqrt{C}$. If $E = 40$ when $C = 25$, find the following.

a E when $C = 49$ **b** C when $E = 10.4$

$E = k\sqrt{C}$

$\begin{cases} E = 40 \\ C = 25 \end{cases}$

$40 = k \times \sqrt{25}$

$40 = 5k$

$\frac{40}{5} = k$

$8 = k$

$E = 8\sqrt{C}$

a) Find E when $C = 49$

$E = 8\sqrt{49}$

$E = 8 \times 7$

$E = 56$

b) Find C when $E = 10.4$

$10.4 = 8\sqrt{C}$

$\frac{10.4}{8} = \sqrt{C}$

$1.3 = \sqrt{C}$

$1.3^2 = C$

$C = 1.69$

5 P is directly proportional to f^3. If $P = 400$ when $f = 10$, find the following.

a P when $f = 4$ **b** f when $P = 50$

$P = kf^3$

$\begin{cases} P = 400 \\ f = 10 \end{cases}$

$400 = k \times 10^3$

$400 = 1000k$

$\frac{400}{1000} = k$

$k = 0.4$

$P = 0.4f^3$

a) Find P when $f=4$

$P = 0.4 \times 4^3$

$P = 0.4 \times 64$

$P = 25.6$

b) Find f when $P=50$

$50 = 0.4 f^3$

$\frac{50}{0.4} = f^3$

$125 = f^3$

$\sqrt[3]{125} = f$

$f = 5$

Homework for Friday

Q6, Q7, Q8, Q9, Q10