Power Functions Exercise 8D

1) $X \sim Po(\lambda)$ $\quad X \sim Po(6.5)$ $\quad H_0 : \lambda = 6.5$, $H_1 : \lambda < 6.5$

Critical Region $x \leq 2$

a) $P(X \leq 2) = 0.0430$ = size of test

b) $P(X \leq 2 \mid X \sim Po(\lambda))$

$$= P(X=0) + P(X=1) + P(X=2)$$

$$= \frac{e^{-\lambda} \times \lambda^0}{0!} + \frac{e^{-\lambda}\lambda}{1} + \frac{e^{-\lambda}\lambda^2}{2!}$$

$$= e^{-\lambda} + e^{-\lambda}\lambda + \tfrac{1}{2}e^{-\lambda}\lambda^2$$

$$= e^{-\lambda}\left(1 + \lambda + \tfrac{1}{2}\lambda^2\right)$$

c) $s = e^{-2}\left(1 + 2 + \tfrac{1}{2}(2)^2\right) = 0.68$

$t = e^{-5}\left(1 + 5 + \tfrac{1}{2}(5)^2\right) = 0.12$

d) Draw graph of power function

e) From graph Power > 0.5 when $\lambda < 2.5$

Also Prob correct > 0.5 when $\lambda = 6.5$

3) $X \sim B(10, 0.4)$

$H_0 : p = 0.4$

$H_1 : p > 0.4$

$Y \sim B(10, 0.5)$

CR $x \geq 8$

$P(Y \leq 7) = 0.9453 =$ Type II error

a) Power $= 1 - 0.9453 = 0.0547$

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b) $Y \sim B(10, 0.8)$

$P(Y \leq 7) = 0.3222$

Power $= 0.6778$

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c) Power increases as p increases away from 0.4

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5) $X \sim Geo(0.15)$

$H_0 : p = 0.15$

$H_1 : p < 0.15$

sig level 1%

$P(X \geq x) = 0.85^{x-1}$

$0.85^{x-1} < 0.01$

$x - 1 > \dfrac{\ln 0.01}{\ln 0.85}$

$x > 28.3 + 1$

$x > 29.3$

CR $x \geq 30$

a) $P(X \geq 30) = 0.85^{29} = 0.00898$

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b) Power function $(1-p)^{29}$