Projectiles - Worked Example

7 The trajectory ABCD of a small stone moving with negligible air resistance is shown in Fig. 7. AD is horizontal and BC is parallel to AD.

The stone is projected from A with speed $40\,\mathrm{m\,s^{-1}}$ at 50° to the horizontal.

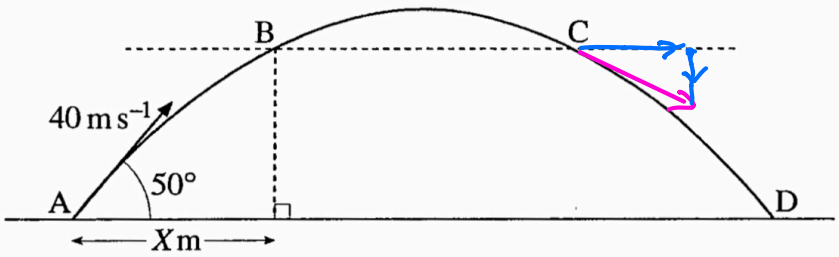


Fig. 7

(i) Write down an expression for the horizontal displacement from A of the stone t seconds after projection. Write down also an expression for the vertical displacement at time t. [3]

(ii) Show that the stone takes 6.253 seconds (to three decimal places) to travel from A to D. Calculate the range of the stone. [5]

You are given that $X = 30$.

(iii) Calculate the time it takes the stone to reach B. Hence determine the time for it to travel from A to C. [4]

(iv) Calculate the direction of the motion of the stone at C. [5]

i) Horizontal Displacement

$x = U\cos\alpha \times t$

$x = 40\cos 50^\circ \times t$

Vertical Displacement

$y = = U\sin\alpha \times t + \frac{1}{2}at^2$

$y = 40\sin 50^\circ \times t - 4.9t^2$

ii) Reaches D when $y = 0$

$0 = 40\sin 50^\circ \times t - 4.9t^2$

$$0 = t(40\sin 50^\circ - 4.9t)$$

Either $\cancel{t = 0}$ or $40\sin 50^\circ - 4.9t = 0$

$$40\sin 50^\circ = 4.9t$$

$$t = \frac{40\sin 50^\circ}{4.9} = 6.25342$$

$$\underline{t = 6.253 \text{ to 3 d.p.}}$$

Horizontal Range $x = 40\cos 50^\circ \times t$

$$x = 40\cos 50^\circ \times 6.253$$

$$x = 160.77 \text{ m}$$

$$\underline{x = 161 \text{ m}}$$

iii) $x = 30$

$\longleftrightarrow$ $x = 40\cos 50^\circ \times t = 30$

$$t = \frac{30}{40\cos 50^\circ}$$

$$t = 1.1668$$

$$\underline{t = 1.17 \text{ s}}$$

time for AB = time for CD = 1.17 s (symmetry)

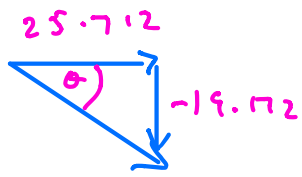
time for AC = time for AD − time for CD

$$= 6.253 - 1.17$$

$$= 5.083 \text{ s}$$

$$= \underline{5.08 \text{ s}}$$

iv



Horizontal Speed at C

$= 40\cos 50^\circ = 25.712\ ms^{-1}$

Vertical speed at C

$V_y = U_y + at$

$V_y = 40\sin 50^\circ - 9.8 \times 5.083$

$V_y = -19.172\ ms^{-1}$

$\theta = \tan^{-1}\left(\frac{19.172}{25.172}\right) = 37.3^\circ$

Direction of motion at C is 37.3° below horizontal