

Composite Functions

$$f(x) = x^2 + 1 \quad g(x) = 2x + 3$$

$$ff(x) = f(x^2+1) = (x^2+1)^2 + 1 = x^4 + 2x^2 + 2$$

$$fg(x) = f(2x+3) = (2x+3)^2 + 1$$

$$fg(3) = (2(3)+3)^2 + 1 = 82$$

Alternatively

$$fg(3)$$

$$\begin{aligned} g(3) &= 2(3) + 3 \\ &= a \end{aligned}$$

$$= f(a)$$

$$= a^2 + 1 = 82$$

Find $g^{-1}(x)$

$$\text{Let } y = g(x) = 2x + 3$$

Swap variables

$$x = 2y + 3$$

$$x - 3 = 2y$$

$$\frac{x-3}{2} = y$$

$$g^{-1}(x) = \frac{x-3}{2}$$

$f(x-a)$ translation by $\begin{pmatrix} a \\ 0 \end{pmatrix}$

$f(x)+b$ translation by $\begin{pmatrix} 0 \\ b \end{pmatrix}$

$f(-x)$ reflection y-axis

$-f(x)$ reflection x-axis