Basic Differentiation

1) $$y = \frac{x^{3/2} + 5x}{\sqrt{x}} = \frac{x^{3/2} + 5x^1}{x^{\frac{1}{2}}} = x + 5x^{\frac{1}{2}}$$

$$\frac{dy}{dx} = 1 + \tfrac{1}{2}\left(5x^{-\frac{1}{2}}\right) = 1 + \tfrac{5}{2}x^{-\frac{1}{2}}$$

$$= 1 + \frac{5}{2\sqrt{x}}$$

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2) $$f(x) = x^3 - 3x$$

Find values of x for which $f(x)$ is a decreasing function

$$f'(x) = 3x^2 - 3$$

Decreasing when $f'(x) < 0$

$$3x^2 - 3 < 0$$

$$3x^2 < 3$$

$$x^2 < \frac{3}{3}$$

$$x^2 < 1$$

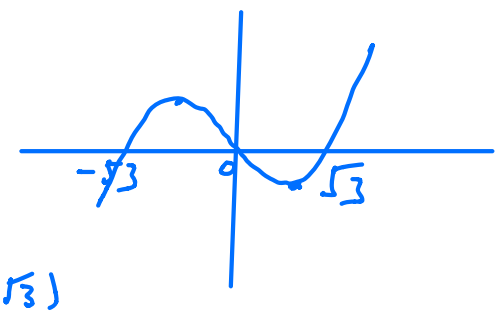
$$-1 < x < 1$$

Sketch function

$$f(x) = x^3 - 3x$$

$$= x(x^2 - 3)$$

$$= x(x + \sqrt{3})(x - \sqrt{3})$$



Find stationary points and confirm their nature

$$f(x) = x^3 - 3x$$

$$f'(x) = 3x^2 - 3$$

At st. pt. $f'(x) = 0$

$$\Rightarrow \quad 3x^2 - 3 = 0$$

$$3x^2 = 3$$

$$x^2 = 1$$

$$x = \pm 1$$

when $x = -1$, $y = (-1)^3 - 3(-1)$

$$= -1 + 3$$

$$= 2$$

st pt at $(-1, 2)$

when $x = 1$ $\quad y = 1^3 - 3(1)$

$$= 1 - 3$$

$$= -2$$

st pt at $(1, -2)$

$$f''(x) = 6x$$

when $x = -1$ $\quad f''(-1) = 6(-1) = -6 < 0 \therefore$ max

Maximum at $(-1, 2)$

when $x=1$ $\quad f''(1) = 6(1) = 6 > 0 \therefore$ min

Minimum at $(1,-2)$

Let $\quad y = x^3 + 5x^2 - 20$

Find the eqn of the normal where $x=2$, and find the area of the triangle this normal makes with the axes

when $x=2$, $\quad y = 2^3 + 5(2)^2 - 20$

$y = 8 + 20 - 20$

$y = 8$

Point $(2,8)$

$$\frac{dy}{dx} = 3x^2 + 10x$$

when $x=2$, $\quad \frac{dy}{dx} = 3(2)^2 + 10(2)$

$= 12 + 20$

$= 32$

$\therefore$ gradient of normal $= -\frac{1}{32}$

$$y - y_1 = m(x - x_1)$$

$$y - 8 = -\frac{1}{32}(x-2)$$

$$y - 8 = -\frac{1}{32}x + \frac{1}{16}$$

$$y = -\frac{1}{32}x + \frac{129}{16}$$

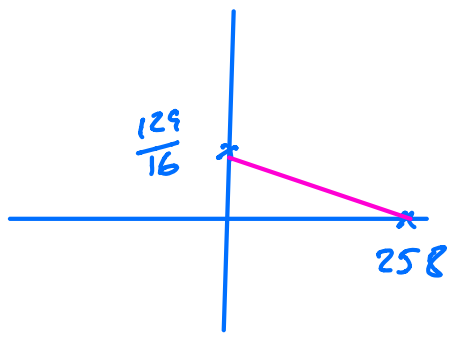
where does this line meet axes?

when $x = 0$ $\quad y = \frac{129}{16}$

when $y = 0$ $\quad 0 = -\frac{1}{32}x + \frac{129}{16}$

$$\frac{1}{32}x = \frac{129}{16}$$

$$x = 258$$



Area of $\triangle$

$= \frac{1}{2}$ base $\times$ height

$= \frac{1}{2} \times 258 \times \frac{129}{16}$

$= 1040$ units^2