

$$\sum_{r=1}^n \frac{1}{r(r+1)} = \frac{n}{n+1}$$

(5)



8. Prove by induction that, for  $n \in \mathbb{Z}^+$ ,

(a)  $f(n) = 5^n + 8n + 3$  is divisible by 4,

(7)

(b)  $\begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}^n = \begin{pmatrix} 2n+1 & -2n \\ 2n & 1-2n \end{pmatrix}$

**(7)**

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

8. (a) Prove by induction that, for any positive integer  $n$ ,

$$\sum_{r=1}^n r^3 = \frac{1}{4} n^2 (n+1)^2 \quad (5)$$

- (b) Using the formulae for  $\sum_{r=1}^n r$  and  $\sum_{r=1}^n r^3$ , show that

$$\sum_{r=1}^n (r^3 + 3r + 2) = \frac{1}{4}n(n+2)(n^2 + 7) \quad (5)$$

- (c) Hence evaluate  $\sum_{r=15}^{25} (r^3 + 3r + 2)$

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9. (a) Prove by induction that

$$\sum_{r=1}^n r^2 = \frac{1}{6}n(n+1)(2n+1) \quad (6)$$

Using the standard results for  $\sum_{r=1}^n r$  and  $\sum_{r=1}^n r^2$ ,

(b) show that

$$\sum_{r=1}^n (r+2)(r+3) = \frac{1}{3}n(n^2 + an + b),$$

where  $a$  and  $b$  are integers to be found.

(c) Hence show that

$$\sum_{r=n+1}^{2n} (r+2)(r+3) = \frac{1}{3}n(7n^2 + 27n + 26) \quad (3)$$



$$u_{n+1} = 4u_n + 2, \quad u_1 = 2$$
$$u_n = \frac{2}{3}(4^n - 1)$$

(5)

9. Prove by induction, that for  $n \in \mathbb{Z}^+$ ,

$$(a) \quad \begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}^n = \begin{pmatrix} 3^n & 0 \\ 3(3^n - 1) & 1 \end{pmatrix}, \quad (6)$$

(b)  $f(n) = 7^{2n-1} + 5$  is divisible by 12. (6)

This image shows a full page of blank, lined paper. It features approximately 20 evenly spaced horizontal gray lines across its entire width, providing a guide for writing. The paper itself is white and has no other markings, text, or illustrations.

6. (a) Prove by induction

$$\sum_{r=1}^n r^3 = \frac{1}{4} n^2 (n+1)^2 \quad (5)$$

(b) Using the result in part (a), show that

$$\sum_{r=1}^n (r^3 - 2) = \frac{1}{4} n(n^3 + 2n^2 + n - 8) \quad (3)$$

(c) Calculate the exact value of  $\sum_{r=20}^{50} (r^3 - 2)$ . (3)



**10.** Prove by induction that, for  $n \in \mathbb{Z}^+$ ,

$f(n) = 2^{2n-1} + 3^{2n-1}$  is divisible by 5.

(6)

[illegible]