Leave blank

4. Prove by induction that, for $n \in \mathbb{Z}^+$,

$$\sum_{r=1}^{n} \frac{1}{r(r+1)} = \frac{n}{n+1}$$

(5)

Leave blank

8. Prove by induction that, for $n \in \mathbb{Z}^+$,

(a) $f(n) = 5^n + 8n + 3$ is divisible by 4,

(7)

(b) $\begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}^n = \begin{pmatrix} 2n+1 & -2n \\ 2n & 1-2n \end{pmatrix}$

(7)

Leave blank

8. (a) Prove by induction that, for any positive integer n,

$$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$

(5)

(b) Using the formulae for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^3$, show that

$$\sum_{r=1}^{n}(r^3 + 3r + 2) = \frac{1}{4}n(n+2)(n^2+7)$$

(5)

(c) Hence evaluate $\sum_{r=15}^{25}(r^3 + 3r + 2)$

(2)

Leave blank

9. (a) Prove by induction that

$$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$$

(6)

Using the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$,

(b) show that

$$\sum_{r=1}^{n} (r+2)(r+3) = \frac{1}{3}n(n^2+an+b),$$

where a and b are integers to be found.

(5)

(c) Hence show that

$$\sum_{r=n+1}^{2n} (r+2)(r+3) = \frac{1}{3}n(7n^2+27n+26)$$

(3)

Leave blank

9. A sequence of numbers $u_1, u_2, u_3, u_4, \ldots$ is defined by

$$u_{n+1} = 4u_n + 2, \quad u_1 = 2$$

Prove by induction that, for $n \in \mathbb{Z}^+$,

$$u_n = \frac{2}{3}\left(4^n - 1\right)$$

(5)

Leave blank

9. Prove by induction, that for $n \in \mathbb{Z}^+$,

(a) $\begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}^n = \begin{pmatrix} 3^n & 0 \\ 3(3^n - 1) & 1 \end{pmatrix}$,

(6)

(b) $f(n) = 7^{2n-1} + 5$ is divisible by 12.

(6)

Leave blank

6. (a) Prove by induction

$$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$

(5)

(b) Using the result in part (a), show that

$$\sum_{r=1}^{n} (r^3 - 2) = \frac{1}{4}n(n^3 + 2n^2 + n - 8)$$

(3)

(c) Calculate the exact value of $\sum_{r=20}^{50} (r^3 - 2)$.

(3)

Leave blank

10. Prove by induction that, for $n \in \mathbb{Z}^+$,

$$f(n) = 2^{2n-1} + 3^{2n-1} \text{ is divisible by 5.}$$

(6)