Algebraic Fractions

1 Simplify each of these.

a $\dfrac{x}{2} + \dfrac{x}{3}$

b $\dfrac{3x}{4} + \dfrac{x}{5}$

c $\dfrac{3x}{4} + \dfrac{2x}{5}$

d $\dfrac{x}{2} + \dfrac{y}{3}$

e $\dfrac{xy}{4} + \dfrac{2}{x}$

f $\dfrac{x+1}{2} + \dfrac{x+2}{3}$

g $\dfrac{2x-1}{2} + \dfrac{3x-1}{4}$

h $\dfrac{x}{5} + \dfrac{2x-1}{3}$

i $\dfrac{x-2}{2} + \dfrac{x+3}{4}$

j $\dfrac{x-4}{5} + \dfrac{2x-3}{2}$

2 Simplify each of these.

a $\dfrac{x}{2} - \dfrac{x}{3}$

b $\dfrac{3x}{4} - \dfrac{x}{5}$

c $\dfrac{3x}{4} - \dfrac{2x}{5}$

d $\dfrac{x}{2} - \dfrac{y}{3}$

e $\dfrac{xy}{4} - \dfrac{2}{y}$

f $\dfrac{x+1}{2} - \dfrac{x+2}{3}$

g $\dfrac{2x+1}{2} - \dfrac{3x+3}{4}$

h $\dfrac{x}{5} - \dfrac{2x+1}{3}$

i $\dfrac{x-2}{2} - \dfrac{x-3}{4}$

j $\dfrac{x-4}{5} - \dfrac{2x-3}{2}$

3 Solve the following equations.

a $\dfrac{x+1}{2} + \dfrac{x+2}{5} = 3$

b $\dfrac{x+2}{4} + \dfrac{x+1}{7} = 3$

c $\dfrac{4x+1}{3} - \dfrac{x+2}{4} = 2$

d $\dfrac{2x-1}{3} + \dfrac{3x+1}{4} = 7$

e $\dfrac{2x+1}{2} - \dfrac{x+1}{7} = 1$

f $\dfrac{3x+1}{5} - \dfrac{5x-1}{7} = 0$

c $\dfrac{3x}{4} + \dfrac{2x}{5}$

f $\dfrac{x+1}{2} + \dfrac{x+2}{3}$

i $\dfrac{x-2}{2} + \dfrac{x+3}{4}$

c) $\dfrac{3x}{4} + \dfrac{2x}{5}$

$= \dfrac{15x + 8x}{20}$

$= \dfrac{23x}{20}$

f) $\dfrac{x+1}{2} + \dfrac{x+2}{3} = \dfrac{3(x+1)+2(x+2)}{6}$

$= \dfrac{3x+3+2x+4}{6} = \dfrac{5x+7}{6}$

i) $\dfrac{x-2}{2} + \dfrac{x+3}{4} = \dfrac{2(x-2)+x+3}{4}$

$= \dfrac{2x-4+x+3}{4} = \dfrac{3x-1}{4}$

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e) $\dfrac{xy}{4} + \dfrac{2}{x} = \dfrac{x^2y+8}{4x}$

a $\dfrac{x}{2} + \dfrac{x}{3}$

d $\dfrac{x}{2} + \dfrac{y}{3}$

g $\dfrac{2x-1}{2} + \dfrac{3x-1}{4}$

a) $\dfrac{x}{2} + \dfrac{x}{3} = \dfrac{3x+2x}{6} = \dfrac{5x}{6}$

d) $\dfrac{x}{2} + \dfrac{y}{3} = \dfrac{3x+2y}{6}$

g) $\dfrac{2x-1}{2} + \dfrac{3x-1}{4} = \dfrac{2(2x-1)+3x-1}{4} = \dfrac{4x-2+3x-1}{4}$

$= \dfrac{7x-3}{4}$

4 Simplify each of these.

a $\dfrac{x}{2} \times \dfrac{x}{3}$

b $\dfrac{2x}{7} \times \dfrac{3y}{4}$

c $\dfrac{4x}{3y} \times \dfrac{2y}{x}$

d $\dfrac{4y^2}{9x} \times \dfrac{3x^2}{2y}$

e $\dfrac{x}{2} \times \dfrac{x-2}{5}$

f $\dfrac{x-3}{15} \times \dfrac{5}{2x-6}$

g $\dfrac{2x+1}{2} \times \dfrac{3x+1}{4}$

h $\dfrac{x}{5} \times \dfrac{2x+1}{3}$

i $\dfrac{x-2}{2} \times \dfrac{4}{x-3}$

j $\dfrac{x-5}{10} \times \dfrac{5}{x^2-5x}$

5 Simplify each of these.

a $\dfrac{x}{2} \div \dfrac{x}{3}$

b $\dfrac{2x}{7} \div \dfrac{4y}{14}$

c $\dfrac{4x}{3y} \div \dfrac{x}{2y}$

d $\dfrac{4y^2}{9x} \div \dfrac{2y}{3x^2}$

e $\dfrac{x}{2} \div \dfrac{x-2}{5}$

f $\dfrac{x-3}{15} \div \dfrac{5}{2x-6}$

g $\dfrac{2x+1}{2} \div \dfrac{4x+2}{4}$

h $\dfrac{x}{6} \div \dfrac{2x^2+x}{3}$

i $\dfrac{x-2}{12} \div \dfrac{4}{x-3}$

j $\dfrac{x-5}{10} \div \dfrac{x^2-5x}{5}$

6 Simplify each of these. Factorise and cancel where appropriate.

a $\dfrac{3x}{4} + \dfrac{x}{4}$

b $\dfrac{3x}{4} - \dfrac{x}{4}$

c $\dfrac{3x}{4} \times \dfrac{x}{4}$

d $\dfrac{3x}{4} \div \dfrac{x}{4}$

e $\dfrac{3x+1}{2} + \dfrac{x-2}{5}$

f $\dfrac{3x+1}{2} - \dfrac{x-2}{5}$

g $\dfrac{3x+1}{2} \times \dfrac{x-2}{5}$

h $\dfrac{x^2-9}{10} \times \dfrac{5}{x-3}$

i $\dfrac{2x+3}{5} \div \dfrac{6x+9}{10}$

j $\dfrac{2x^2}{9} - \dfrac{2y^2}{3}$

7 Show that each algebraic fraction simplifies to the given expression.

a $\dfrac{2}{x+1} + \dfrac{5}{x+2} = 3$ simplifies to $3x^2 + 2x - 3 = 0$

b $\dfrac{4}{x-2} + \dfrac{7}{x+1} = 3$ simplifies to $3x^2 - 14x + 4 = 0$

c $\dfrac{3}{4x+1} - \dfrac{4}{x+2} = 2$ simplifies to $8x^2 + 31x + 2 = 0$

d $\dfrac{2}{2x-1} - \dfrac{6}{x+1} = 11$ simplifies to $22x^2 + 21x - 19 = 0$

e $\dfrac{3}{2x-1} - \dfrac{4}{3x-1} = 1$ simplifies to $x^2 - x = 0$

c $\dfrac{4x}{3y} \times \dfrac{2y}{x} \quad = \quad \dfrac{8}{3}$

f $\dfrac{x-3}{15} \times \dfrac{5}{2x-6}$

i $\dfrac{x-2}{2} \times \dfrac{4}{x-3}$

f) $\dfrac{(x-3)^1}{15_3} \times \dfrac{5^1}{2(x-3)} \quad = \quad \dfrac{1}{6}$

i) $\dfrac{(x-2)}{2} \times \dfrac{4^2}{(x-3)} \quad = \quad \dfrac{2(x-2)}{x-3}$

$$\text{or} \qquad \dfrac{2x-4}{x-3}$$

b $\dfrac{2x}{7} \times \dfrac{3y}{4} \quad = \quad \dfrac{6xy}{28}$

e $\dfrac{x}{2} \times \dfrac{x-2}{5} \quad = \quad \dfrac{x^2-2x}{10}$

h $\dfrac{x}{5} \times \dfrac{2x+1}{3} \quad = \quad \dfrac{2x^2+x}{15}$

c $\dfrac{4x}{3y} \div \dfrac{x}{2y}$

f $\dfrac{x-3}{15} \div \dfrac{5}{2x-6}$

i $\dfrac{x-2}{12} \div \dfrac{4}{x-3}$

c) $\dfrac{4x^1}{3y_1} \times \dfrac{2y^1}{x_1} = \dfrac{8}{3}$

f) $\dfrac{(x-3)}{15} \times \dfrac{(2x-6)}{5} = \dfrac{2x^2 - 6x - 6x + 18}{75}$

$$= \dfrac{2x^2 - 12x + 18}{75}$$