Venn Hwk 2 Solutions

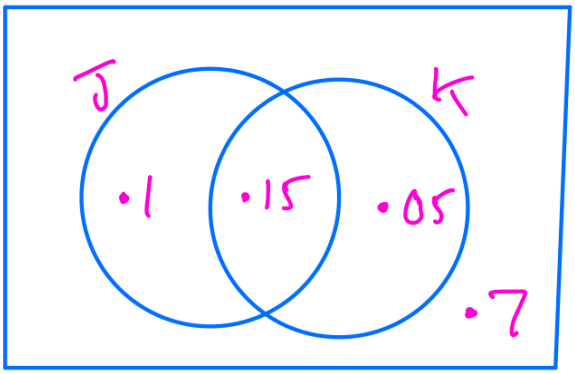
Q4.

Jake and Kamil are sometimes late for school.
The events J and K are defined as follows

J = the event that Jake is late for school

K = the event that Jake is late for school

$P(J) = 0.25$, $P(J \cap K) = 0.15$ and $P(J' \cap K') = 0.7$



On a randomly selected day, find the probability that

(a) at least one of Jake or Kamil are late for school,

$P(J \cup K) = 0.3$ **(1)**

(b) Kamil is late for school.

$P(K) = 0.2$ **(2)**

Given that Jake is late for school,

(c) find the probability that Kamil is late. $\frac{\cdot 15}{\cdot 25} = 0.6$

(3)

The teacher suspects that Jake being late for school and Kamil being late for school are linked in some way.

(d) Determine whether or not J and K are statistically independent.

$P(J) \times P(K) = \cdot 25 \times \cdot 2 = 0.05 \neq 0.15 = P(J \cap K)$

$\therefore$ not independent **(2)**

(e) Comment on the teacher's suspicion in the light of your calculation in (d).

Teacher's suspicion correct **(1)**

(Total 9 marks)

Q7.

(a) Given that P(A) = a and P(B) = b express P($A \cup B$) in terms of a and b when

(i) A and B are mutually exclusive,

$P(A \cup B) = P(A) + P(B) = a + b$

(ii) A and B are independent.

$P(A \cap B) = P(A) \times P(B) = ab$

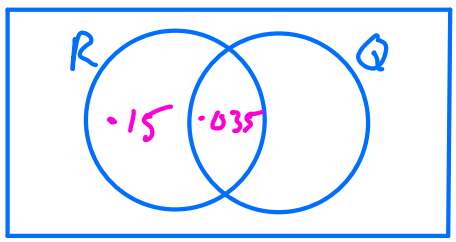
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= a + b - ab$

(2)

Two events R and Q are such that

$P(R \cap Q') = 0.15$, $P(Q) = 0.35$ and $P(R|Q) = 0.1$



Find the value of

(b) P($R \cup Q$),

(1)

(c) P($R \cap Q$),

(2)

(d) P(R).

(2)

(Total 7 marks)

b) $P(R \cup Q) = P(Q) + P(R \cap Q')$

$= 0.35 + 0.15$

$= 0.5$

c) $P(R|Q) = \frac{P(R \cap Q)}{P(Q)}$ $\quad 0.1 = \frac{P(R \cap Q)}{0.35}$

$P(R \cap Q) = 0.035$

d) $P(R) = 0.15 + 0.035 = 0.185$

Q8.

Given that

$$P(A) = 0.35 \text{ , } P(B) = 0.45 \text{ and } P(A \cap B) = 0.13$$

find $(A \cup B)$ $= P(A) + P(B) - P(A \cap B)$

$= 0.35 + 0.45 - 0.13 = 0.67$ **(2)**

(b) $P(A' \mid B')$

(2)

The event C has $P(C) = 0.20$

The events A and C are mutually exclusive and the events B and C are independent.

(c) Find $P(B \cap C)$

(2)

(d) Draw a Venn diagram to illustrate the events A, B and C and the probabilities for each region.

(4)

Tom invites Avisha to play a game with these dice.

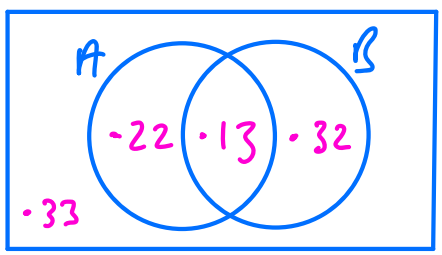
(e) Find $P([B \cup C]')$

(2)

(Total 12 marks)

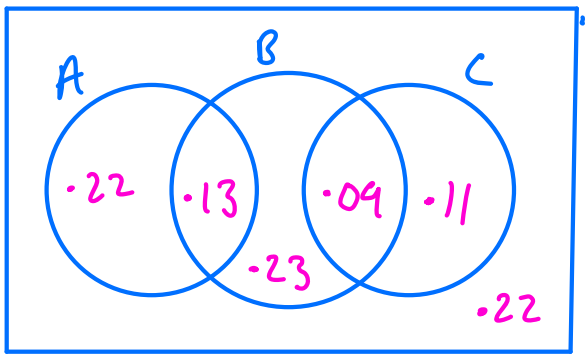
b) $P(A' \mid B') = \dfrac{P(A' \cap B')}{P(B')}$

$= \dfrac{0.33}{0.55} = 0.6$



c) $P(B \cap C) = P(B) \times P(C) = 0.45 \times 0.2 = 0.09$

d)



e) $P([B \cup C]') = 0.22 + 0.22 = 0.44$

Q9.

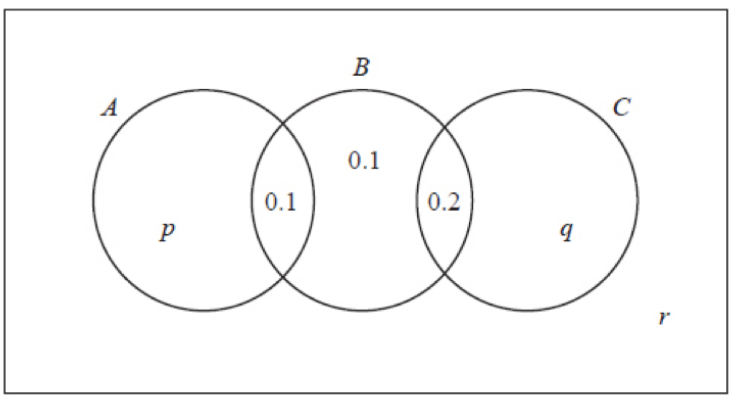


Figure 1

The Venn diagram in Figure 1 shows three events A, B and C and the probabilities associated with each region of B. The constants p, q and r each represent probabilities associated with the three separate regions outside B.

The events A and B are independent.

(a) Find the value of p.

(3)

Given that $P(B \mid C) = \frac{5}{11}$

(b) find the value of q and the value of r.

(4)

(c) Find $P(A \cup C \mid B)$.

(2)

(Total 9 marks)

a) $P(A) \times P(B) = P(A \cap B)$

$(p + 0.1) \times 0.4 = 0.1$

$0.4p + 0.04 = 0.1$

$0.4p = 0.06$

$p = \frac{0.06}{0.4} = 0.15$

---

b) $P(B|C) = \frac{5}{11} = \frac{P(B \cap C)}{P(C)} = \frac{0.2}{0.2 + q}$

$5(0.2 + q) = 0.2 \times 11$

$1 + 5q = 2.2$

$5q = 1.2$

$q = 0.24$

$r = 1 - 0.4 - p - q$

$r = 0.6 - 0.15 - 0.24$

$r = 0.21$

---

c) $P([A \cup C]|B) = \frac{0.3}{0.4} = 0.75$

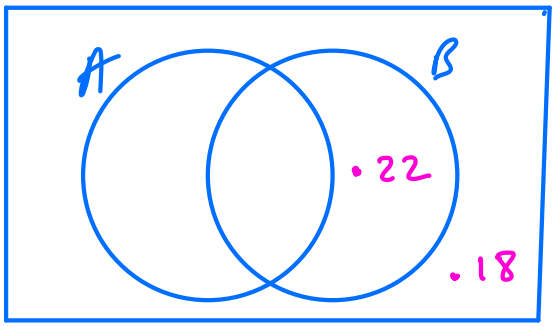
Q10.

For the events A and B,

$$P(A' \cap B) = 0.22 \text{ and } P(A' \cap B') = 0.18$$

(a) Find P(A).

$= 1 - 0.4 = 0.6$



(1)

(b) Find P($A \cup B$).

$= 0.6 + 0.22$

$= 0.82$

(1)

Given that P($A \mid B$) = 0.6

$P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

(c) find P($A \cap B$).

(3)

(d) Determine whether or not A and B are independent.

(2)

(Total 7 marks)

c) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore \quad P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$P(A \cap B) = 0.6 + P(B) - 0.82$

$= P(B) - 0.22$

$P(A \mid B) = \dfrac{P(B) - 0.22}{P(B)} = 0.6$

$P(B) - 0.22 = 0.6\,P(B)$

$0.4\,P(B) = 0.22$

$P(B) = \dfrac{0.22}{0.4} = 0.55$

$\Rightarrow P(A \cap B) = 0.55 - 0.22 = 0.33$

d) $P(A) \times P(B) = 0.6 \times 0.55 = 0.33$

$= P(A \cap B)$

Yes A and B are independent

Q11.

A and B are two events such that

$$P(B) = \tfrac{1}{2} \quad P(A \mid B) = \tfrac{2}{5} \quad P(A \cup B) = \tfrac{13}{20}$$

(a) Find $P(A \cap B)$.

(2)

(b) Draw a Venn diagram to show the events A, B and all the associated probabilities.

(3)

Find

(c) $P(A)$

(1)

(d) $P(B \mid A)$

(2)

(e) $P(A' \cap B)$

(1)

(Total 9 marks)

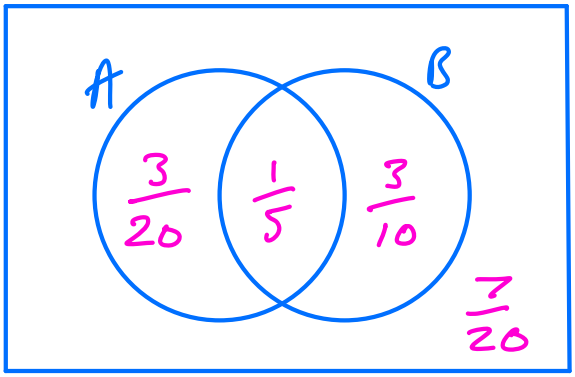
a) $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

$P(A \cap B) = P(A \mid B) \times P(B)$

$= \dfrac{2}{5} \times \dfrac{1}{2} = \dfrac{1}{5}$

---

b)



c) $P(A) = \frac{3}{20} + \frac{1}{5} = \frac{7}{20}$

d) $P(B \mid A) = \frac{P(B \cap A)}{P(A)} = \frac{\frac{1}{5}}{\frac{7}{20}}$

$= \frac{1}{5} \times \frac{20}{7} = \frac{4}{7}$

e) $P(A' \cap B) = \frac{3}{10}$

Q12.

A college has 80 students in Year 12.

20 students study Biology
28 students study Chemistry
30 students study Physics

7 students study both Biology and Chemistry
11 students study both Chemistry and Physics
5 students study both Physics and Biology
3 students study all 3 of these subjects

(a) Draw a Venn diagram to represent this information.

(5)

A Year 12 student at the college is selected at random.

(b) Find the probability that the student studies Chemistry but not Biology or Physics.

(1)

(c) Find the probability that the student studies Chemistry or Physics or both.

(2)

Given that the student studies Chemistry or Physics or both,

(d) find the probability that the student does not study Biology.

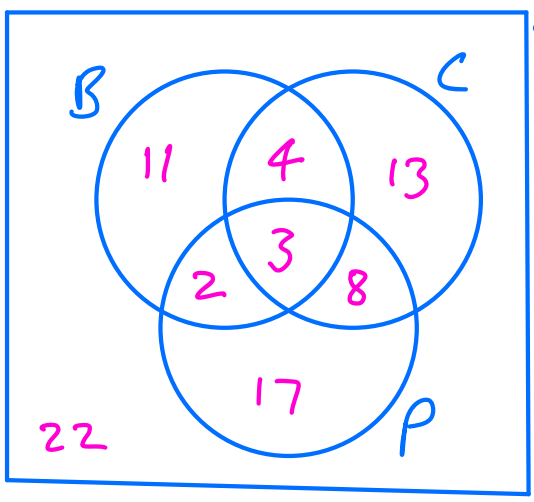
(2)

(e) Determine whether studying Biology and studying Chemistry are statistically independent.

(3)

(Total for question = 13 marks)

a)



b) $\frac{13}{80}$

c) $\frac{47}{80}$

d) $\frac{13+8+17}{47} = \frac{38}{47}$

$P(B) \times P(C)$

$= \frac{20}{80} \times \frac{28}{80} = \frac{7}{80}$

$P(B \cap C) = \frac{7}{80}$

$P(B) \times P(C) = P(B \cap C)$

So yes they are independent

Q14.

The following shows the results of a wine tasting survey of 100 people.

96 like wine A,
93 like wine B,
96 like wine C,
92 like A and B,
91 like B and C,
93 like A and C,
90 like all three wines.

(a) Draw a Venn Diagram to represent these data.

(6)

Find the probability that a randomly selected person from the survey likes

(b) none of the three wines,

(1)

(c) wine A but not wine B,

(2)

(d) any wine in the survey except wine C,

(2)

(e) exactly two of the three kinds of wine.

(2)

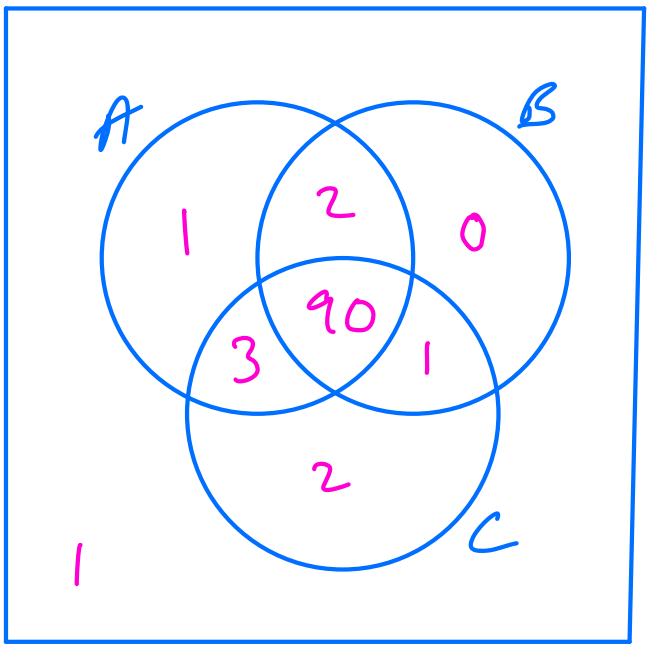
Given that a person from the survey likes wine A,

(f) find the probability that the person likes wine C.

(3)

(Total 16 marks)

a)



b) $\frac{1}{100}$

c) $\frac{4}{100}$

d) $\frac{3}{100}$

e) $\frac{1+2+3}{100} = \frac{6}{100}$

f) $P(A \setminus C) = \frac{93}{96}$

Q15.

A survey of the reading habits of some students revealed that, on a regular basis, 25% read quality newspapers, 45% read tabloid newspapers and 40% do not read newspapers at all.

(a) Find the proportion of students who read both quality and tabloid newspapers.

(3)

(b) Draw a Venn diagram to represent this information.

(3)

A student is selected at random. Given that this student reads newspapers on a regular basis,

(c) find the probability that this student only reads quality newspapers.

(3)

(Total 9 marks)

a) $P(Q) = 0.25$

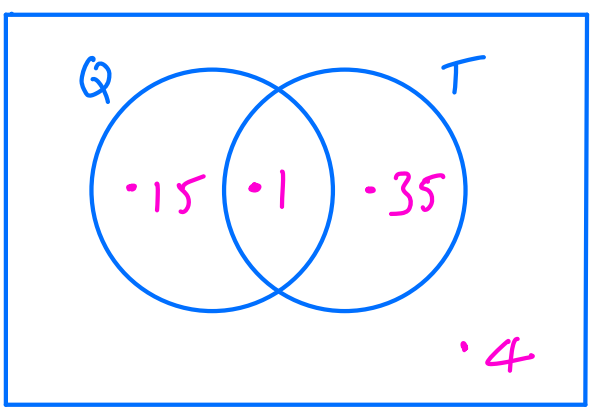
$P(T) = 0.45$

$P(N) = 0.40$

1.10

$P(Q \cap T) = 0.1$

b)



c) $P(Q \cap T' \mid Q \cup T)$

$= \frac{0.15}{0.6}$

$= 0.25$

Q16.

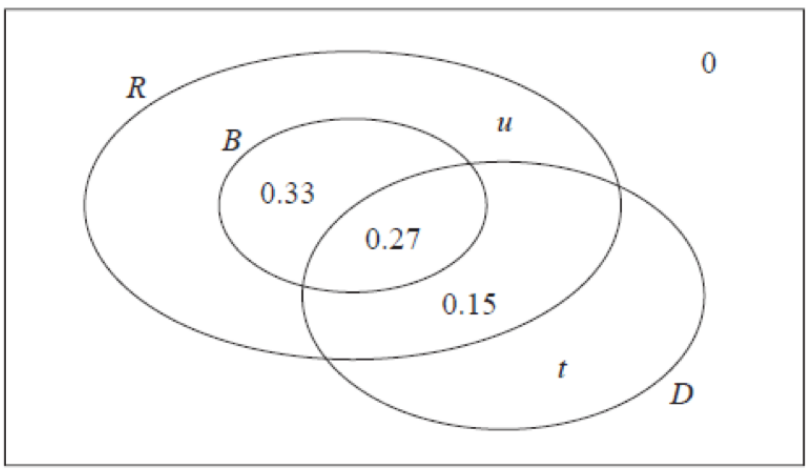
The Venn diagram shows the probabilities of customer bookings at Harry's hotel.

R is the event that a customer books a room

B is the event that a customer books breakfast

D is the event that a customer books dinner

u and t are probabilities.



(a) Write down the probability that a customer books breakfast but does not book a room.

(1)

Given that the events B and D are independent

(b) find the value of t

(4)

(c) hence find the value of u

(2)

$P(B) = 0.6 \quad P(D) = 0.42 + t$

$P(B \cap D) = 0.27$

$\Rightarrow 0.6(0.42 + t) = 0.27$

$0.42 + t = \frac{0.27}{0.6}$

$t = 0.45 - 0.42$

$t = 0.03$

c) $u = 1 - (0.33 + 0.27 + 0.15 + 0.03) = 0.22$

(d) Find

(i) $P(D|R \cap B)$

(ii) $P(D|R \cap B')$

i) $R \cap B = 0.6 \quad P(D \mid R \cap B) = \frac{0.27}{0.6} = 0.45$

ii) $R \cap B' = 0.15 + 0.22 = 0.37$

$P(D \mid R \cap B') = \frac{0.15}{0.37} = \frac{15}{37}$

(4)

A coach load of 77 customers arrive at Harry's hotel.

Of these 77 customers

40 have booked a room and breakfast

37 have booked a room without breakfast

(e) Estimate how many of these 77 customers will book dinner.

(2)

e) $40 \times 0.45 + 37 \times \frac{15}{37}$

$= 18 + 15$

$= 33$

(Total for question = 13 marks)