

Prime Factors, HCFs, LCMs

2, 3, 5, 7, 11, 13

All integers greater than 1 can be factorised into prime factors

Ex1 24
$$\begin{array}{r} 2 \overline{)24} \\ 2 \overline{)12} \\ 2 \overline{)6} \\ 3 \overline{)3} \\ 1 \end{array}$$

$$24 = 2 \times 2 \times 2 \times 3$$
$$24 = 2^3 \times 3$$

Ex2 78
$$\begin{array}{r} 2 \overline{)78} \\ 3 \overline{)39} \\ 13 \overline{)13} \\ 1 \end{array}$$

$$78 = 2 \times 3 \times 13$$

Ex3 100
$$\begin{array}{r} 2 \overline{)100} \\ 2 \overline{)50} \\ 5 \overline{)25} \\ 5 \overline{)5} \\ 1 \end{array}$$

$$100 = 2 \times 2 \times 5 \times 5$$
$$100 = 2^2 \times 5^2$$

Exercise Write these numbers as a product of their prime factors

1) 84 2) 60 3) 35 4) 90 5) 120

1) 84
$$\begin{array}{r} 2 \overline{)84} \\ 2 \overline{)42} \\ 3 \overline{)21} \\ 7 \overline{)7} \\ 1 \end{array}$$

$$84 = 2 \times 2 \times 3 \times 7$$
$$84 = 2^2 \times 3 \times 7$$

$$2) \quad 60 \quad \begin{array}{r} 2 \overline{)60} \\ 2 \overline{)30} \\ 3 \overline{)15} \\ 5 \overline{)5} \\ 1 \end{array}$$

$$60 = 2 \times 2 \times 3 \times 5$$

$$60 = 2^2 \times 3 \times 5$$

$$3) \quad 35 \quad \begin{array}{r} 5 \overline{)35} \\ 7 \overline{)7} \\ 1 \end{array}$$

$$35 = 5 \times 7$$

$$4) \quad 90 \quad \begin{array}{r} 2 \overline{)90} \\ 3 \overline{)45} \\ 3 \overline{)15} \\ 5 \overline{)5} \\ 1 \end{array}$$

$$90 = 2 \times 3 \times 3 \times 5$$

$$90 = 2 \times 3^2 \times 5$$

$$5) \quad 120 \quad \begin{array}{r} 2 \overline{)120} \\ 2 \overline{)60} \\ 2 \overline{)30} \\ 3 \overline{)15} \\ 5 \overline{)5} \\ 1 \end{array}$$

$$120 = 2 \times 2 \times 2 \times 3 \times 5$$

$$120 = 2^3 \times 3 \times 5$$

Highest Common Factors HCFs

The HCF of two integers A and B is the biggest number which divides into both of them.

With small numbers we can often identify the HCF by inspection

$$\text{Ex1} \quad 8 \quad \text{and} \quad 12$$

$$\text{HCF} = 4$$

$$\text{Ex 2} \quad 15 \text{ and } 20 \quad \text{HCF} = 5$$

$$\text{Ex 3} \quad 9 \text{ and } 14 \quad \text{HCF} = 1$$

We can use prime factors to find an HCF

$$\text{Ex 1} \quad 60 \text{ and } 96$$

$$\begin{array}{r} 2 \overline{)60} \\ 2 \overline{)30} \\ 3 \overline{)15} \\ 5 \overline{)5} \\ 1 \end{array}$$

$$\begin{array}{r} 2 \overline{)96} \\ 2 \overline{)48} \\ 2 \overline{)24} \\ 2 \overline{)12} \\ 2 \overline{)6} \\ 3 \overline{)3} \\ 1 \end{array}$$

$$60 = 2 \times 2 \times 3 \times 5$$

$$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$$

$$\text{HCF} = 2 \times 2 \times 3$$

$$\underline{\text{HCF} = 12}$$

$$\text{Ex 2} \quad 75 \text{ and } 125$$

$$\begin{array}{r} 3 \overline{)75} \\ 5 \overline{)25} \\ 5 \overline{)5} \\ 1 \end{array}$$

$$\begin{array}{r} 5 \overline{)125} \\ 5 \overline{)25} \\ 5 \overline{)5} \\ 1 \end{array}$$

$$75 = 3 \times 5 \times 5$$

$$125 = 5 \times 5 \times 5$$

$$\text{HCF} = 5 \times 5$$

$$\underline{\text{HCF} = 25}$$

Exercise Find the HCF of

1) 48 and 32

3) 70 and 105

2) 144 and 96

4) 56 and 84

$$\begin{array}{r}
 1) \quad 2 \overline{)48} \\
 \quad 2 \overline{)24} \\
 \quad \quad 2 \overline{)12} \\
 \quad \quad \quad 2 \overline{)6} \\
 \quad \quad \quad \quad 3 \overline{)3} \\
 \quad \quad \quad \quad \quad 1
 \end{array}$$

$$\begin{array}{r}
 2 \overline{)32} \\
 \quad 2 \overline{)16} \\
 \quad \quad 2 \overline{)8} \\
 \quad \quad \quad 2 \overline{)4} \\
 \quad \quad \quad \quad 2 \overline{)2} \\
 \quad \quad \quad \quad \quad 1
 \end{array}$$

$$48 = 2 \times 2 \times 2 \times 2 \times 3$$

$$32 = 2 \times 2 \times 2 \times 2 \times 2$$

$$\text{HCF} = 2 \times 2 \times 2 \times 2$$

$$\underline{\text{HCF} = 16}$$

$$\begin{array}{r}
 2) \quad 2 \overline{)144} \\
 \quad 2 \overline{)72} \\
 \quad \quad 2 \overline{)36} \\
 \quad \quad \quad 2 \overline{)18} \\
 \quad \quad \quad \quad 3 \overline{)9} \\
 \quad \quad \quad \quad \quad 3 \overline{)3} \\
 \quad \quad \quad \quad \quad \quad 1
 \end{array}$$

$$\begin{array}{r}
 2) \quad 2 \overline{)96} \\
 \quad 2 \overline{)48} \\
 \quad \quad 2 \overline{)24} \\
 \quad \quad \quad 2 \overline{)12} \\
 \quad \quad \quad \quad 2 \overline{)6} \\
 \quad \quad \quad \quad \quad 3 \overline{)3} \\
 \quad \quad \quad \quad \quad \quad 1
 \end{array}$$

$$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$$

$$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$$

$$\text{HCF} = 2 \times 2 \times 2 \times 2 \times 3$$

$$\underline{\text{HCF} = 48}$$

$$3) \quad \text{HCF} = 35$$

$$4) \quad \text{HCF} = 28$$

LOWEST COMMON MULTIPLES LCMs

The LCM of two integers A and B is the lowest number that A and B will divide into exactly.

Examples with small numbers

1) 8 and 12 \rightarrow LCM = 24

8, 16, 24, 32, 40
12, 24

2) 10 and 15 \rightarrow LCM = 30

3) 48 and 72

48, 96, 144, 192
72, 144

LCM = 144

Exercise Find LCMs

1) 45 and 60 LCM = 180

2) 27 and 54 LCM = 54

3) 64 and 96 LCM = 192

4) 45 90 135 180
60 120 180 LCM = 180

3) 64 128 192 256 320 384
96 192 288 384
