Mean and standard Deviation

See Exercise on Estimating the Mean after these lesson notes.

A class is made up of 10 girls and 20 boys. In a test the boys average 54 marks and the girls average 64 marks. Calculate the mean mark for the class.

Girls $10 \times 64 = 640$ marks

Boys $20 \times 54 = 1080$ marks

Total marks 1720 marks

Mean for class $= \frac{1720}{30} = 57.3$

In a class of 24 students the mean test mark was 48. The 15 girls had a mean mark of 45. What was the mean mark of the boys?

Class total marks $= 24 \times 48 = 1152$ marks

Girls total marks $= 15 \times 45 = 675$

Boys total marks 477

Boys mean mark $= \frac{477}{9} = 53$

Number of boys $= 24 - 15 = 9$

Standard Deviation

$$\text{Variance} = \frac{\Sigma (x - \bar{x})^2}{n}$$

$$\text{standard deviation} = \sqrt{\frac{\Sigma (x - \bar{x})^2}{n}}$$

$$= \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2}$$

The second formula is easier to use in practice

The units of standard deviation are same units of the data. The units of variance are not meaningful.

In a class the 12 boys have a mean marks of 50 and a standard deviation of 5. The 18 girls have a mean mark of 80 and a standard deviation of 3

Find the mean and standard deviation for the whole class

Boys $\bar{y} = 50$ $\sqrt{\frac{\Sigma y^2}{n_y} - \bar{y}^2} = 5$ $n_y = 12$

Girls $\bar{x} = 80$ $\sqrt{\frac{\Sigma x^2}{n_x} - \bar{x}^2} = 3$ $n_x = 18$

Mean for class $= \bar{z}$ $= \frac{\text{Total Marks}}{\text{Total People}} = \frac{12\times 50 + 18\times 80}{30}$

$= 68$

Find s.d. for class

$$= \sqrt{\frac{\Sigma x^2 + \Sigma y^2}{30} - \bar{z}^2}$$

Boys $s.d = \sqrt{\sum \frac{y^2}{n} - \bar{y}^2}$

$$5 = \sqrt{\frac{\Sigma y^2}{12} - 50^2}$$

$$25 = \frac{\Sigma y^2}{12} - 50^2$$

$$12(25 + 50^2) = \Sigma y^2 = 30\,300$$

Girls $sd = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2}$

$$3 = \sqrt{\frac{\Sigma x^2}{18} - 80^2}$$

$$9 = \frac{\Sigma x^2}{18} - 80^2$$

$$18(9+80^2) = \Sigma x^2 = 115362$$

$$\text{Class s.d.} = \sqrt{\frac{115362 + 30300}{30} - 68^2}$$

$$= 15.2$$
