Logs and Exponentials

$\log ab = \log a + \log b$

$\log \frac{a}{b} = \log a - \log b$

$\log a^b = b \log a$

$\log_a a = 1$

$\log_a 1 = 0$

1. (a) Given that

$$2\log_3(x-5) - \log_3(2x-13) = 1,$$

show that $x^2 - 16x + 64 = 0$.

(5)

(b) Hence, or otherwise, solve $2\log_3(x-5) - \log_3(2x-13) = 1$.

(2)

(Total 7 marks)

a)

$$2\log_3(x-5) - \log_3(2x-13) = 1$$

$$\log_3(x-5)^2 - \log_3(2x-13) = 1$$

$$\log_3\left(\frac{(x-5)^2}{2x-13}\right) = 1$$

$$\frac{(x-5)^2}{2x-13} = 3^1$$

$$x^2 - 10x + 25 = 3(2x-13)$$

$$x^2 - 10x + 25 = 6x - 39$$

$$x^2 - 10x + 25 - 6x + 39 = 0$$

$$x^2 - 16x + 64 = 0$$

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b)

$$x^2 - 16x + 64 = 0$$

$$(x-8)(x-8) = 0$$

$$x = 8$$

2. (a) Find the positive value of x such that

$$\log x\ 64 = 2$$

(2)

(b) Solve for x

$$\log_2(11 - 6x) = 2\log_2(x - 1) + 3$$

(6)
(Total 8 marks)

a)

$$x^2 = 64$$

$$x = 8$$

b)

$$\log_2(11-6x) = 2\log_2(x-1) + 3$$

$$\log_2(11-6x) - \log_2(x-1)^2 = 3$$

$$\log_2\left(\frac{11-6x}{(x-1)^2}\right) = 3$$

$$\frac{11-6x}{(x-1)^2} = 2^3 = 8$$

$$11 - 6x = 8(x^2 - 2x + 1)$$

$$11 - 6x = 8x^2 - 16x + 8$$

$$0 = 8x^2 - 16x + 8 + 6x - 11$$

$$0 = 8x^2 - 10x - 3$$

$$0 = 8x^2 + 2x - 12x - 3$$

$$0 = 2x(4x+1) - 3(4x+1)$$

$$0 = (2x-3)(4x+1)$$

$$x = \frac{3}{2} \quad \text{or} \quad x = -\frac{1}{4}$$

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3. Given that $0 < x < 4$ and

$$\log_5(4-x) - 2\log_5 x = 1,$$

find the value of x.

(Total 6 marks)

$$\log_5(4-x) - \log_5 x^2 = 1$$

$$\log_5\left(\frac{4-x}{x^2}\right) = 1$$

$$\frac{4-x}{x^2} = 5^1 = 5$$

$$4 - x = 5x^2$$

$$0 = 5x^2 + x - 4$$

$$0 = (5x-4)(x+1)$$

$$x = \frac{4}{5} \qquad \cancel{x = -1}$$

4. Given that a and b are positive constants, solve the simultaneous equations

$$a = 3b, \quad (1)$$

$$\log_3 a + \log_3 b = 2. \quad (2)$$

Give your answers as exact numbers.

(Total 6 marks)

From (2) $\log_3(ab) = 2$

$ab = 3^2 = 9$

$\Rightarrow b = \frac{9}{a}$

Sub in (1)

$a = 3 \times \frac{9}{a}$

$a^2 = 27$

$a = 3\sqrt{3}$

$b = \frac{9}{3\sqrt{3}}$

$b = \sqrt{3}$

7. Find, giving your answer to 3 significant figures where appropriate, the value of x for which

(a) $3^x = 5$,

(3)

(b) $\log_2 (2x + 1) - \log_2 x = 2$.

(4)

(Total 7 marks)

a) $3^x = 5$

$\log_{10} 3^x = \log_{10} 5$

$x \log_{10} 3 = \log_{10} 5$

$x = \dfrac{\log_{10} 5}{\log_{10} 3}$

$x = 1.46$ to 3 s.f.

b) $\log_2(2x+1) - \log_2 x = 2$

$\log_2\left(\dfrac{2x+1}{x}\right) = 2$

$\dfrac{2x+1}{x} = 2^2$

$2x + 1 = 4x$

$1 = 4x - 2x$

$1 = 2x$

$x = \frac{1}{2}$

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5. (i) Write down the value of $\log_6 36$.

(1)

(ii) Express $2 \log_a 3 + \log_a 11$ as a single logarithm to base a.

(3)

(Total 4 marks)

i) $\log_6 36 = 2$ (because $36 = 6^2$)

ii) $2\log_a 3 + \log_a 11$

$= \log_a(3^2) + \log_a 11 = \log_a(9 \times 11) = \log_a 99$

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6. Solve

(a) $5^x = 8$, giving your answers to 3 significant figures,

(3)

(b) $\log_2(x + 1) - \log_2 x = \log_2 7$.

(3)
(Total 6 marks)

a) $5^x = 8$

$\ln 5^x = \ln 8$

$x \ln 5 = \ln 8$

$x = \frac{\ln 8}{\ln 5} = 1.29$

b) $\log_2(x+1) - \log_2(x) = \log_2 7$

$\log_2\left(\frac{x+1}{x}\right) = \log_2 7$

$\frac{x+1}{x} = 7$

$x + 1 = 7x$

$1 = 7x - x$

$1 = 6x$

$\frac{1}{6} = x$ $\qquad$ $x = \frac{1}{6}$
