

$$y = \arctan(\sinh(x))$$

(a) Show that $\frac{d^3y}{dx^3} = \frac{dy}{dx} - 2\left(\frac{dy}{dx}\right)^3$ (7)

(b) Hence find $\frac{d^5y}{dx^5}$ in terms of $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ and $\frac{d^3y}{dx^3}$ (4)

(c) Find the Maclaurin series for y , in ascending powers of x , up to and including the term in x^5

(3)

b) $\frac{d^3y}{dx^3} = \frac{dy}{dx} - 2\left(\frac{dy}{dx}\right)^3$

$$\frac{d^4y}{dx^4} = \frac{d^2y}{dx^2} - 6\left(\frac{dy}{dx}\right)^2\left(\frac{d^2y}{dx^2}\right)$$

$$\frac{d^5y}{dx^5} = \frac{d^3y}{dx^3} - 6\left(\frac{dy}{dx}\right)^2\left(\frac{d^3y}{dx^3}\right) - 12\left(\frac{dy}{dx}\right)\left(\frac{d^2y}{dx^2}\right)^2$$

a) $y = \arctan(\sinh x)$

Let $u = \sinh x$

$$y = \arctan u$$

$$\frac{du}{dx} = \cosh x$$

$$\frac{dy}{dx} = \frac{1}{1+u^2} \cdot \frac{du}{dx}$$

$$\frac{dy}{dx} = \frac{1}{1+\sinh^2 x} \cdot \cosh x$$

$$\frac{dy}{dx} = \frac{1}{\cosh x} = (\cosh x)^{-1}$$

$$\frac{d^2y}{dx^2} = -1(\cosh x)^{-2}(\sinh x) = \frac{-\sinh x}{(\cosh x)^2}$$

$$\frac{d^3y}{dx^3} = \frac{(\cosh x)^2(-\cosh x) - (-\sinh x)2(\cosh x)\sinh x}{\cosh^4 x}$$

$$\frac{d^3y}{dx^3} = \frac{-\cosh^2 x + 2 \sinh^2 x}{\cosh^3 x}$$

$$= \frac{-\cosh^2 x + 2(\cosh^2 x - 1)}{\cosh^3 x}$$

$$= \frac{\cosh^2 x - 2}{\cosh^3 x}$$

$$= \frac{1}{\cosh x} - \frac{2}{\cosh^3 x} = \frac{dy}{dx} - 2 \left(\frac{dy}{dx} \right)^3$$

$$f(0) = 0$$

$$f'(0) = 1$$

$$f''(0) = 0$$

$$f'''(0) = -1$$

$$f^{(4)}(0) = 0$$

$$f^{(5)}(0) = 5$$

$$y = 0 + 1x + \frac{0x^2}{2!} - \frac{1x^3}{3!} + \frac{0x^4}{4!} + \frac{5x^5}{5!}$$

$$y = x - \frac{1}{6}x^3 + \frac{1}{24}x^5$$

4. (a) Using the identity $zz^* = |z|^2$, or otherwise, show that if w is any root of unity then

$$|w - z|^2 = 5 - 2(w + w^*) \quad (3)$$

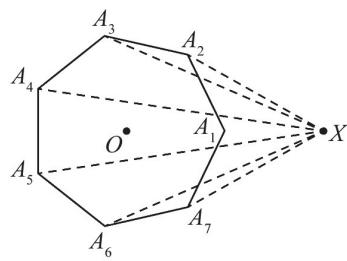


Figure 1

Figure 1 shows a regular heptagon $A_1A_2A_3A_4A_5A_6A_7$ whose vertices all lie on the unit circle with centre at the origin O and A_1 at $(1, 0)$. The point X lies in the same plane as the heptagon and has coordinates $(2, 0)$.

Using the result given in part (a),

$$(b) \text{ find } \sum_{i=1}^7 (XA_i)^2 \quad (4)$$

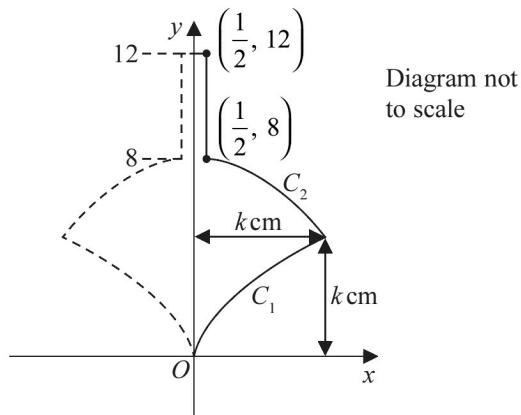
$$\begin{aligned} a) \quad |w - z|^2 &= (w - z)(w - z)^* \\ &= (w - z)(w^* - z) \\ &= ww^* - zw^* - zw + 4 \\ &= |w|^2 - 2(w^* + w) + 4 \\ &= 1 - 2(w^* + w) + 4 \\ &= 5 - 2(w^* + w) \end{aligned}$$

$$\begin{aligned} b) \quad XA_i &= |w - z| \quad \text{where } w \text{ is a } 7^{\text{th}} \text{ root} \\ (XA_i)^2 &= |w - z|^2 \quad \text{of unity} \end{aligned}$$

$$\begin{aligned} \sum_{i=1}^7 (XA_i)^2 &= 5 - 2(A_1 + A_1^*) \\ &\quad + 5 - 2(A_2 + A_2^*) \\ &\quad + \\ &\quad + 5 - 2(A_7 + A_7^*) \\ &= 35 - 2[\sum A_i + \sum A_i^*] \end{aligned}$$

$$= 35 - 2[0 + 0] = 35$$

7.



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Figure 2

Figure 2 shows a sketch of the cross-section of a design for a child's spinning top. The top is formed by rotating the region bounded by the y -axis, the curve C_1 , the curve C_2 , the line with equation $x = \frac{1}{2}$ and the line with equation $y = 12$, through 360° about the y -axis.

The curve C_1 has equation

$$y = k^{\frac{2}{3}}x^{\frac{1}{3}} \quad 0 \leq x \leq k$$

and the curve C_2 has equation

$$y = \frac{32k^2 - k - (32 - 4k)x^2}{4k^2 - 1} \quad \frac{1}{2} \leq x \leq k$$

(a) Show that $\int_k^8 ((4k^2 - 1)y - (32k^2 - k)) dy = \frac{1}{2}(8 - k)(4k^3 - 32k^2 + k - 8)$ (3)

Hence find

(b) the value of k that gives the maximum value for the volume of the spinning top, (9)

(c) the maximum volume of the spinning top. (3)

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$$|Vol| = 4\pi \left(\frac{1}{2}\right)^2 + \pi \int_0^k x_1^2 dy + \pi \int_k^8 x_2^2 dy$$