1 (a) Express

$$5\sinh x + \cosh x$$

in the form $Ae^{x} + Be^{-x}$, where A and B are integers. *(2 marks)*

(b) Solve the equation

$$5\sinh x + \cosh x + 5 = 0$$

giving your answer in the form $\ln a$, where a is a rational number. *(4 marks)*

a) $\frac{5}{2}(e^{x}-e^{-x}) + \frac{1}{2}(e^{x}+e^{-x})$

$= 3e^{x} - 2e^{-x}$

b) $3e^{x} - 2e^{-x} + 5 = 0$

$\times e^{x}$ $\quad 3e^{2x} + 5e^{x} - 2 = 0$

$(3e^{x} - 1)(e^{x} + 2) = 0$

Either $e^{x} = \frac{1}{3}$ $\qquad \cancel{e^{x} = -2}$ no solution

$x = \ln\frac{1}{3}$

4 (a) Sketch the graph of $y = \tanh x$. *(2 marks)*

(b) Given that $u = \tanh x$, use the definitions of $\sinh x$ and $\cosh x$ in terms of e^x and e^{-x} to show that

$$x = \frac{1}{2}\ln\left(\frac{1+u}{1-u}\right)$$ *(6 marks)*

(c) (i) Show that the equation

$$3\,\text{sech}^2 x + 7\tanh x = 5$$

can be written as

$$3\tanh^2 x - 7\tanh x + 2 = 0$$ *(2 marks)*

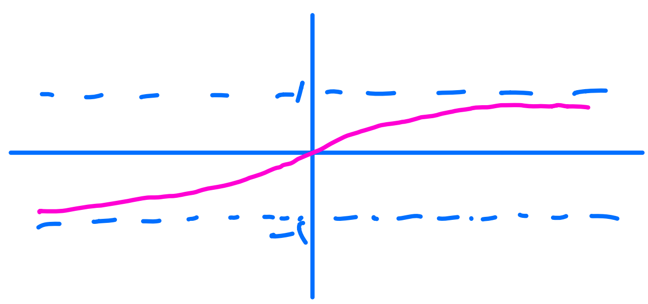
(ii) Show that the equation

$$3\tanh^2 x - 7\tanh x + 2 = 0$$

has only one solution for x.

Find this solution in the form $\frac{1}{2}\ln a$, where a is an integer. *(5 marks)*

a



b)

$$u = \tanh x = \frac{\sinh x}{\cosh x}$$

$$u = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$u(e^x + e^{-x}) = e^x - e^{-x}$$

$$ue^x + ue^{-x} = e^x - e^{-x}$$

$\times e^x$

$$ue^{2x} + u = e^{2x} - 1$$

$$(u-1)e^{2x} = -1-u$$

$$e^{2x} = \frac{-1-u}{u-1}$$

$$e^{2x} = \frac{1+u}{1-u}$$

$$2x = \ln\left(\frac{1+u}{1-u}\right)$$

$$x = \frac{1}{2}\ln\left(\frac{1+u}{1-u}\right) = \operatorname{artanh} u$$

(c) (i) Show that the equation

$$3\operatorname{sech}^2 x + 7\tanh x = 5$$

can be written as

$$3\tanh^2 x - 7\tanh x + 2 = 0$$ *(2 marks)*

Aside

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$1 - \tanh^2 x = \operatorname{sech}^2 x$$

$$3\operatorname{sech}^2 x + 7\tanh x = 5$$

$$3(1-\tanh^2 x) + 7\tanh x - 5 = 0$$

$$3 - 3\tanh^2 x + 7\tanh x - 5 = 0$$

$$0 = 3\tanh^2 x - 7\tanh x + 2$$

(ii) Show that the equation

$$3\tanh^2 x - 7\tanh x + 2 = 0$$

has only one solution for x.

Find this solution in the form $\frac{1}{2}\ln a$, where a is an integer. *(5 marks)*

$$(3\tanh x - 1)(\tanh x - 2) = 0$$

$$\Rightarrow \tanh x = \frac{1}{3} \quad \text{or} \quad \cancel{\tanh x = 2}$$

no solution since $-1 < \tanh x < 1$

$$x = \tanh^{-1}\left(\frac{1}{3}\right)$$

$$x = \frac{1}{2}\ln\left(\frac{1+\frac{1}{3}}{1-\frac{1}{3}}\right)$$

$$x = \frac{1}{2}\ln 2$$

1 (a) Show that

$$9\sinh x - \cosh x = 4e^{x} - 5e^{-x}$$ *(2 marks)*

(b) Given that

$$9\sinh x - \cosh x = 8$$

find the exact value of $\tanh x$. *(7 marks)*

5 (a) Using the identities

$$\cosh^2 t - \sinh^2 t = 1, \quad \tanh t = \frac{\sinh t}{\cosh t} \quad \text{and} \quad \operatorname{sech} t = \frac{1}{\cosh t}$$

show that:

(i) $\tanh^2 t + \operatorname{sech}^2 t = 1$; *(2 marks)*

(ii) $\frac{d}{dt}(\tanh t) = \operatorname{sech}^2 t$; *(3 marks)*

(iii) $\frac{d}{dt}(\operatorname{sech} t) = -\operatorname{sech} t \tanh t$. *(3 marks)*

(b) A curve C is given parametrically by

$$x = \operatorname{sech} t, \quad y = 4 - \tanh t$$

(i) Show that the arc length, s, of C between the points where $t = 0$ and $t = \frac{1}{2}\ln 3$ is given by

$$s = \int_0^{\frac{1}{2}\ln 3} \operatorname{sech} t \, dt$$ *(4 marks)*

(ii) Using the substitution $u = e^{t}$, find the exact value of s. *(6 marks)*

4 (a) Prove that the curve

$$y = 12\cosh x - 8\sinh x - x$$

has exactly one stationary point. *(7 marks)*

(b) Given that the coordinates of this stationary point are (a, b), show that $a + b = 9$. *(4 marks)*

For Q5 use the arc length parametric integration formula on Page 21 of formula book

2 (a) Use the definitions of $\cosh\theta$ and $\sinh\theta$ in terms of e^{θ} to show that

$$\cosh x \cosh y - \sinh x \sinh y = \cosh(x - y)$$ *(4 marks)*

(b) It is given that x satisfies the equation

$$\cosh(x - \ln 2) = \sinh x$$

(i) Show that $\tanh x = \frac{5}{7}$. *(4 marks)*

(ii) Express x in the form $\frac{1}{2}\ln a$. *(2 marks)*

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1 (a) Show, by means of a sketch, that the curves with equations

$$y = \sinh x$$

and $$y = \operatorname{sech} x$$

have exactly one point of intersection. *(4 marks)*

(b) Find the x-coordinate of this point of intersection, giving your answer in the form $a \ln b$. *(4 marks)*

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3 A curve has cartesian equation

$$y = \frac{1}{2}\ln(\tanh x)$$

(a) Show that

$$\frac{dy}{dx} = \frac{1}{\sinh 2x}$$ *(4 marks)*

(b) The points A and B on the curve have x-coordinates $\ln 2$ and $\ln 4$ respectively. Find the arc length AB, giving your answer in the form $p \ln q$, where p and q are rational numbers. *(8 marks)*

QUESTION