Algebraic Methods Mixed Exercise 7

1 a) $\dfrac{3x^4 - 21x}{3x} = \dfrac{3x(x^3 - 7)}{3x} = x^3 - 7$

1 b) $\dfrac{x^2 - 2x - 24}{x^2 - 7x + 6} = \dfrac{(x+4)\cancel{(x-6)}}{(x-1)\cancel{(x-6)}} = \dfrac{x+4}{x-1}$

1 c) $\dfrac{2x^2 + 7x - 4}{2x^2 + 9x + 4} = \dfrac{(2x-1)\cancel{(x+4)}}{(2x+1)\cancel{(x+4)}} = \dfrac{2x-1}{2x+1}$

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2)

$$
\begin{array}{r|l}
 & 3x^2 + 5 \\
\hline
x+4 & 3x^3 + 12x^2 + 5x + 20 \\
 & \underline{3x^3 + 12x^2} \\
 & \qquad\qquad + 5x + 20 \\
 & \qquad\qquad \underline{+ 5x + 20}
\end{array}
$$

Ans $= 3x^2 + 5$

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3) $\dfrac{2x^3 + 3x + 5}{x + 1}$

$$
\begin{array}{r|l}
 & 2x^2 - 2x + 5 \\
\hline
x+1 & 2x^3 \qquad + 3x + 5 \\
 & \underline{2x^3 + 2x^2} \\
 & \qquad -2x^2 + 3x \\
 & \qquad \underline{-2x^2 - 2x} \\
 & \qquad\qquad + 5x + 5 \\
 & \qquad\qquad \underline{+ 5x + 5}
\end{array}
$$

Ans $= 2x^2 - 2x + 5$

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4) $f(x) = 2x^3 - 2x^2 - 17x + 15$

$f(3) = 2(3)^3 - 2(3)^2 - 17(3) + 15$

$= 54 - 18 - 51 + 15$

$= 0$

By factor theorem $(x-3)$ is a factor of $f(x)$

$$
\begin{array}{r|l}
 & 2x^2 + 4x - 5 \\
\hline
x-3 & 2x^3 - 2x^2 - 17x + 15 \\
 & 2x^3 - 6x^2 \\
\hline
 & \quad 4x^2 - 17x \\
 & \quad 4x^2 - 12x \\
\hline
 & \qquad -5x + 15 \\
 & \qquad -5x + 15 \\
\hline
\end{array}
$$

Ans $= (x-3)(2x^2 + 4x - 5)$

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5) $f(x) = x^3 + 4x^2 - 3x - 18$

$f(2) = 2^3 + 4(2)^2 - 3(2) - 18$

$= 8 + 16 - 6 - 18$

$= 0$

By factor theorem $(x-2)$ is a factor of $f(x)$

$$
\begin{array}{r|l}
 & x^2 + 6x + 9 \\
\hline
x-2 & x^3 + 4x^2 - 3x - 18 \\
 & x^3 - 2x^2 \\
\hline
 & \quad 6x^2 - 3x \\
 & \quad 6x^2 - 12x \\
\hline
 & \qquad +9x - 18 \\
 & \qquad +9x - 18 \\
\hline
\end{array}
$$

$x^2 + 6x + 9 = (x+3)^2$

Ans $= (x-2)(x+3)^2$

6) $f(x) = 2x^3 + 3x^2 - 18x + 8$

$f(2) = 2(2)^3 + 3(2)^2 - 18(2) + 8$

$= 16 + 12 - 36 + 8$

$= 0$

$\therefore (x-2)$ is a factor of $f(x)$

$$
\begin{array}{r|l}
 & 2x^2 + 7x - 4 \\
\hline
x-2 & 2x^3 + 3x^2 - 18x + 8 \\
 & \underline{2x^3 - 4x^2} \\
 & \quad 7x^2 - 18x \\
 & \quad \underline{7x^2 - 14x} \\
 & \qquad -4x + 8 \\
 & \qquad \underline{-4x + 8}
\end{array}
$$

$f(x) = (x-2)(2x^2 + 7x - 4)$

$= (x-2)(2x-1)(x+4)$

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7) $f(x) = x^3 - 3x^2 + kx - 10$

$f(2) = 2^3 - 3(2)^2 + 2k - 10 = 0$

$8 - 12 + 2k - 10 = 0$

$2k = 14$

$k = 7$

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8) $f(x) = 2x^2 + px + q$

$f(-3) = 2(-3)^2 - 3p + q = 0$

$18 - 3p + q = 0$ ①

$$f(4) = 2(4)^2 + 4p + q = 21$$

$$32 + 4p + q = 21$$

$$11 + 4p + q = 0 \quad (2)$$

$(1) - (2)$

$$7 - 7p = 0$$

$$7 = 7p$$

$$\underline{p = 1}$$

Sub for p in (2)

$$11 + 4 + q = 0$$

$$\underline{q = -15}$$

8b)

$$f(x) = 2x^2 + x - 15$$

$$= (2x - 5)(x + 3)$$
