Radians Mixed Exercise

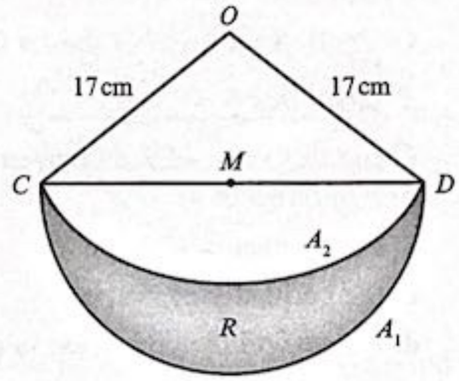
Mixed exercise 5

(P) 1 Triangle ABC is such that $AB = 5$ cm, $AC = 10$ cm and $\angle ABC = 90°$.
An arc of a circle, centre A and radius 5 cm, cuts AC at D.

a State, in radians, the value of $\angle BAC$.

b Calculate the area of the region enclosed by BC, DC and the arc BD.

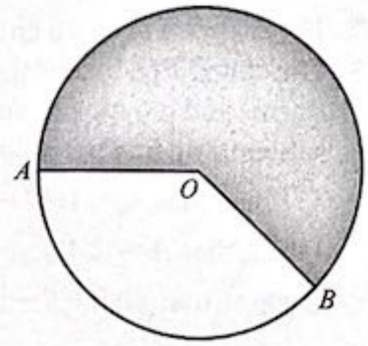
(E/P) 2 The diagram shows the triangle OCD with $OC = OD = 17$ cm and $CD = 30$ cm. The midpoint of CD is M. A semicircular arc A_1, with centre M is drawn, with CD as diameter.
A circular arc A_2 with centre O and radius 17 cm, is drawn from C to D. The shaded region R is bounded by the arcs A_1 and A_2. Calculate, giving answers to 2 decimal places:



a the area of the triangle OCD **(4 marks)**

b the area of the shaded region R. **(5 marks)**

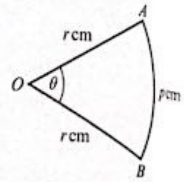
(E/P) 3 The diagram shows a circle, centre O, of radius 6 cm.
The points A and B are on the circumference of the circle.
The area of the shaded major sector is 80 cm^2.
Given that $\angle AOB = \theta$ radians, where $0 < \theta < \pi$, calculate:



a the value, to 3 decimal places, of θ **(3 marks)**

b the length in cm, to 2 decimal places, of the minor arc AB. **(2 marks)**

(E/P) 4 The diagram shows a sector OAB of a circle, centre O and radius r cm. The length of the arc AB is p cm and $\angle AOB$ is θ radians.



a Find θ in terms of p and r. **(2 marks)**

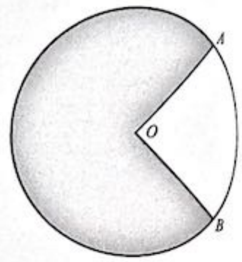
b Deduce that the area of the sector is $\frac{1}{2}pr$ cm^2. **(2 marks)**

Given that $r = 4.7$ and $p = 5.3$, where each has been measured to 1 decimal place, find, giving your answer to 3 decimal places:

c the least possible value of the area of the sector **(2 marks)**

d the range of possible values of θ. **(3 marks)**

(E) 5 The diagram shows a circle centre O and radius 5 cm. The length of the minor arc AB is 6.4 cm.



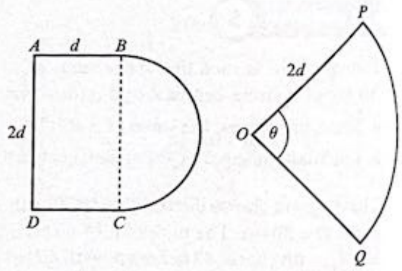
a Calculate, in radians, the size of the acute angle AOB. **(2 marks)**

The area of the minor sector AOB is R_1 cm^2 and the area of the shaded major sector is R_2 cm^2.

b Calculate the value of R_1. **(2 marks)**

c Calculate $R_1 : R_2$ in the form $1 : p$, giving the value of p to 3 significant figures. **(3 marks)**

(E/P) 6 The diagrams show the cross-sections of two drawer handles. Shape X is a rectangle $ABCD$ joined to a semicircle with BC as diameter. The length $AB = d$ cm and $BC = 2d$ cm. Shape Y is a sector OPQ of a circle with centre O and radius $2d$ cm. Angle POQ is θ radians.



Shape X Shape Y

Given that the areas of shapes X and Y are equal,

a prove that $\theta = 1 + \frac{\pi}{4}$ **(5 marks)**

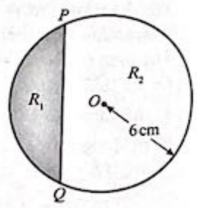
Using this value of θ, and given that $d = 3$, find in terms of π:

b the perimeter of shape X **(3 marks)**

c the perimeter of shape Y. **(3 marks)**

d Hence find the difference, in mm, between the perimeters of shapes X and Y. **(1 mark)**

(E/P) 7 The diagram shows a circle centre O and radius 6 cm. The chord PQ divides the circle into a minor segment R_1 of area A_1 cm^2 and a major segment R_2 of area A_2 cm^2. The chord PQ subtends an angle θ radians at O.



a Show that $A_1 = 18(\theta - \sin\theta)$. **(2 marks)**

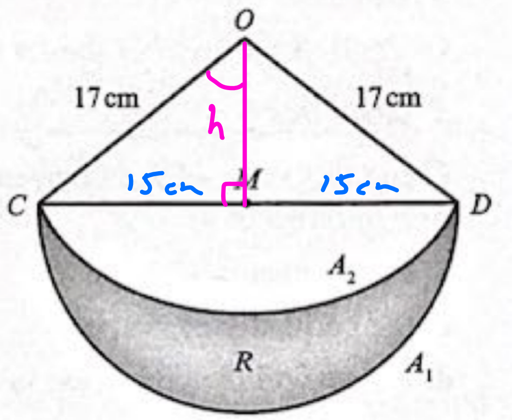
Given that $A_2 = 3A_1$,

b show that $\sin\theta = \theta - \frac{\pi}{2}$ **(4 marks)**

(E/P) 2 The diagram shows the triangle OCD with $OC = OD = 17\text{cm}$ and $CD = 30\text{cm}$. The midpoint of CD is M. A semicircular arc A_1, with centre M is drawn, with CD as diameter. A circular arc A_2 with centre O and radius 17 cm, is drawn from C to D. The shaded region R is bounded by the arcs A_1 and A_2. Calculate, giving answers to 2 decimal places:

a the area of the triangle OCD **(4 marks)**

b the area of the shaded region R. **(5 marks)**



a) By Pythagoras $h^2 + 15^2 = 17^2$

$h^2 = 17^2 - 15^2$

$h^2 = 289 - 225 = 64$

$h = \sqrt{64}$ $\quad h = 8\text{cm}$

Area of $\triangle OCD = \frac{1}{2}$ base $\times$ height

$= \frac{1}{2} \times 30 \times 8 = 120\text{cm}^2$

b) $R =$ Semi-Circle on CD $-$ Segment on CD

$= \dfrac{\pi \times 15^2}{2} - (\text{sector} - \triangle)$

Find $\angle COM = \sin^{-1}\frac{15}{17} = 1.08$

$\angle COD = 1.08 \times 2 = 2.161678$

Area of sector $= \frac{1}{2}r^2\theta = \frac{1}{2} \times 17^2 \times 2.161678$

$= 312.3625$

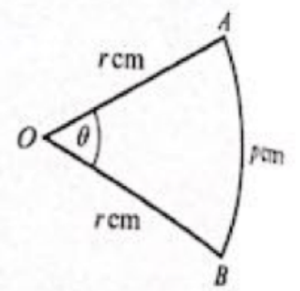
$$R = \frac{\pi \times 15^2}{2} - (312.3625 - 120)$$

$$R = 161 \text{ cm}^2$$

E/P 4 The diagram shows a sector OAB of a circle, centre O and radius r cm. The length of the arc AB is p cm and $\angle AOB$ is θ radians.

a Find θ in terms of p and r. **(2 marks)**

b Deduce that the area of the sector is $\frac{1}{2}pr$ cm². **(2 marks)**



Given that $r = 4.7$ and $p = 5.3$, where each has been measured to 1 decimal place, find, giving your answer to 3 decimal places:

c the least possible value of the area of the sector **(2 marks)**

d the range of possible values of θ. **(3 marks)**

a) Arc length $= r\theta$

$$p = r\theta$$

b) Area $= \frac{1}{2}r^2\theta$

$$= \frac{1}{2}r(r\theta)$$

$$= \frac{1}{2}rp \text{ cm}^2$$

c)

$4.65\text{cm} \le r < 4.75\text{cm}$

$5.25\text{cm} \le p < 5.35\text{cm}$

$$\text{Area} = \frac{1}{2} rp$$

$$\text{min Area} = \frac{1}{2} \times 4.65 \times 5.25$$

$$= 12.20625$$

$$= 12.206 \text{ cm}^2 \text{ to 3 d.p.}$$

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d) From (a) $\theta = \frac{p}{r}$

$$\text{Max } \theta = \frac{p_{max}}{r_{min}} = \frac{5.35}{4.65}$$

$$= 1.151 \text{ to 3dp}$$

$$\text{Min } \theta = \frac{p_{min}}{r_{max}} = \frac{5.25}{4.75}$$

$$= 1.105 \text{ to 3dp}$$

$1.105 \text{ radians} \leq \theta < 1.151 \text{ radians}$