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6. The function f is defined by

$$f: x \mapsto \frac{3-2x}{x-5}, \quad x \in \mathbb{R}, \quad x \neq 5$$

- (a) Find $f^{-1}(x)$.

(3)

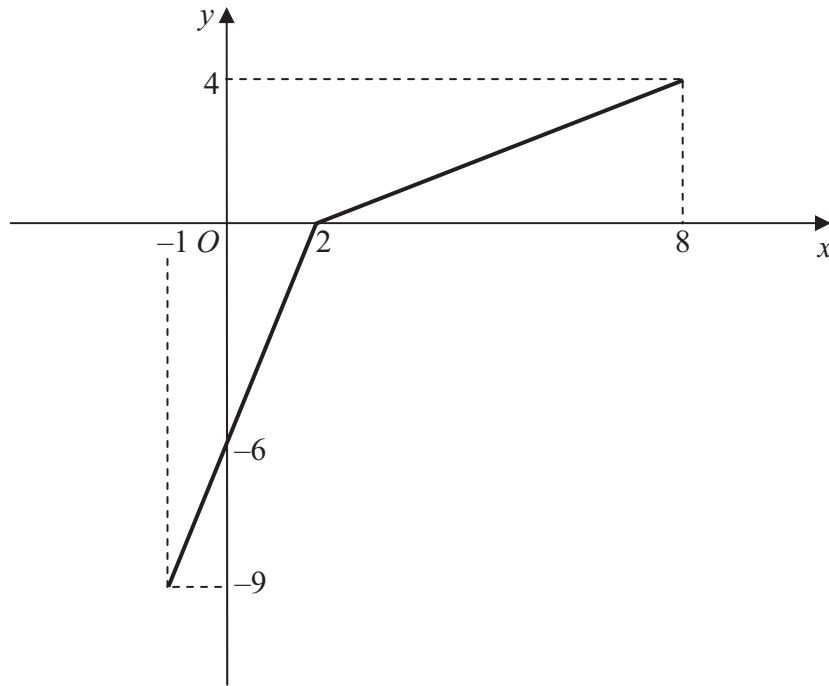


Figure 2

The function g has domain $-1 \leq x \leq 8$, and is linear from $(-1, -9)$ to $(2, 0)$ and from $(2, 0)$ to $(8, 4)$. Figure 2 shows a sketch of the graph of $y = g(x)$.

- (b) Write down the range of g .

(1)

- (c) Find $gg(2)$.

(2)

- (d) Find $fg(8)$.

(2)

- (e) On separate diagrams, sketch the graph with equation

(i) $y = |g(x)|$,

(ii) $y = g^{-1}(x)$.

Show on each sketch the coordinates of each point at which the graph meets or cuts the axes.

(4)

- (f) State the domain of the inverse function g^{-1} .

(1)



Question 6 continued

$$\text{Let } y = \frac{3 - 2x}{x - 5}$$

a)

$$\text{Swap variables} \quad x = \frac{3 - 2y}{y - 5}$$

$$x(y - 5) = 3 - 2y$$

$$xy - 5x = 3 - 2y$$

$$xy + 2y = 3 + 5x$$

$$y(x + 2) = 3 + 5x$$

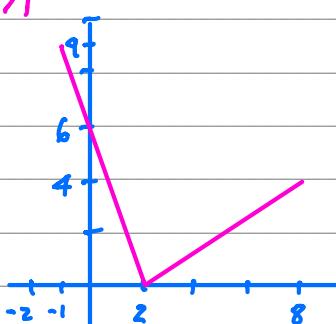
$$y = \frac{5x + 3}{x + 2}$$

b) Range of $g(x)$ $-9 \leq g(x) \leq 4$

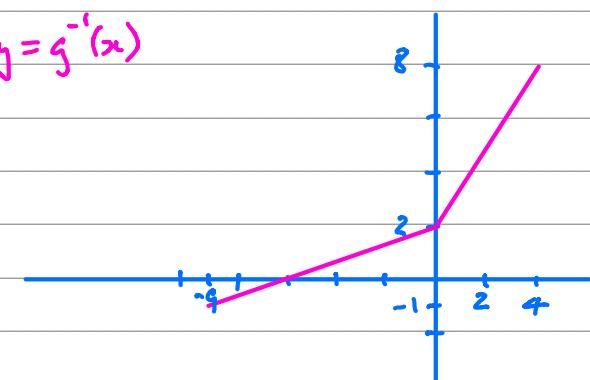
c) $gg(2) = g(0) = -6$

d) $fg(8) = f(4) = \frac{3 - 2(4)}{4 - 5} = 5$

e) $y = |g(x)|$



$y = g^{-1}(x)$



f) Domain of g^{-1} $-9 \leq x \leq 4$



3.

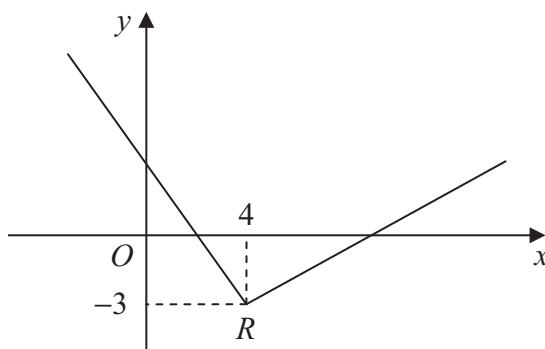
**Figure 1**

Figure 1 shows part of the graph of $y = f(x)$, $x \in \mathbb{R}$.

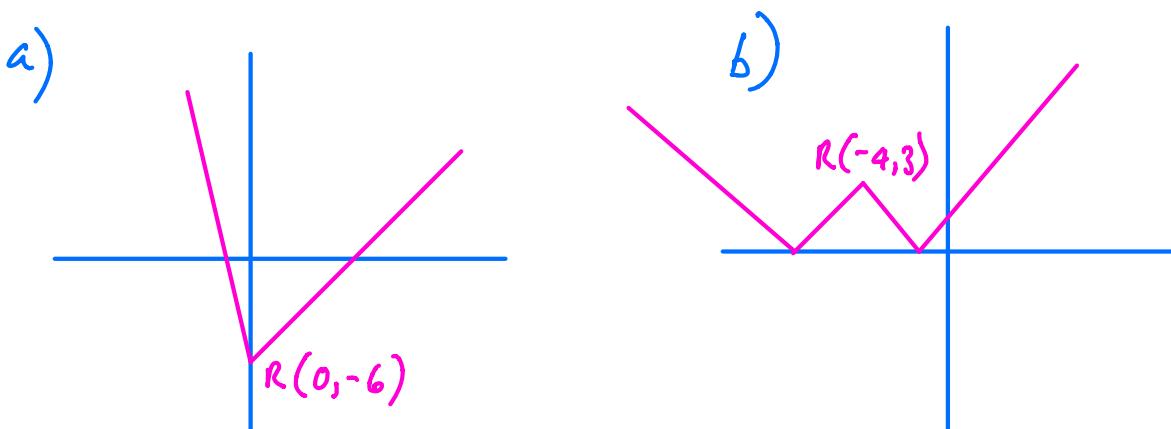
The graph consists of two line segments that meet at the point $R(4, -3)$, as shown in Figure 1.

Sketch, on separate diagrams, the graphs of

(a) $y = 2f(x+4)$, (3)

(b) $y = |f(-x)|$. (3)

On each diagram, show the coordinates of the point corresponding to R .



4. The function f is defined by

$$f : x \mapsto 4 - \ln(x+2), \quad x \in \mathbb{R}, \quad x \geq -1$$

(a) Find $f^{-1}(x)$. (3)

(b) Find the domain of f^{-1} . (1)

The function g is defined by

$$g : x \mapsto e^{x^2} - 2, \quad x \in \mathbb{R}$$

(c) Find $fg(x)$, giving your answer in its simplest form. (3)

(d) Find the range of fg . (1)

a) Let $y = 4 - \ln(x+2)$

swap variables $x = 4 - \ln(y+2)$

$$\ln(y+2) = 4-x$$

$$y+2 = e^{4-x}$$

$$y = e^{4-x} - 2$$

$$f^{-1}(x) = e^{4-x} - 2$$

b) Domain of $f^{-1}(x)$ $x \in \mathbb{R}, \quad x \leq 4$

c) $f(x) = 4 - \ln(x+2) \quad g(x) = e^{x^2} - 2$

$$fg(x) = f(e^{x^2} - 2)$$

$$= 4 - \ln(e^{x^2} - 2 + 2)$$

$$= 4 - \ln(e^{x^2}) \quad = 4 - x^2$$



Question 6 continued

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d) Range of $f \circ g$ $f \circ g(x) \in \mathbb{R}, f \circ g(x) \leq 4$



2.

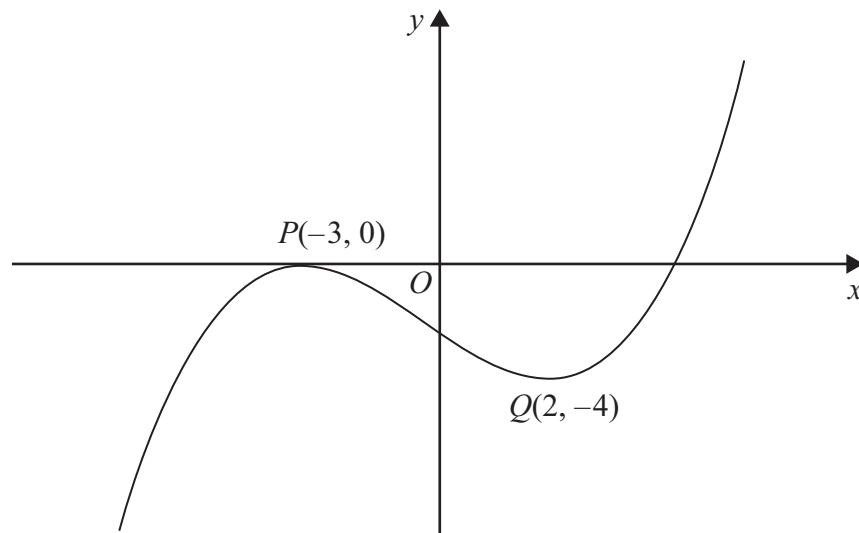
**Figure 1**

Figure 1 shows the graph of equation $y = f(x)$.

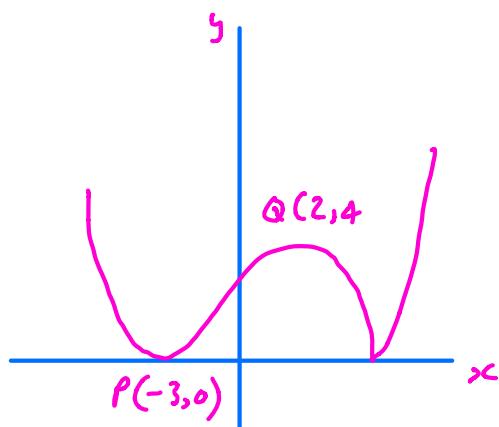
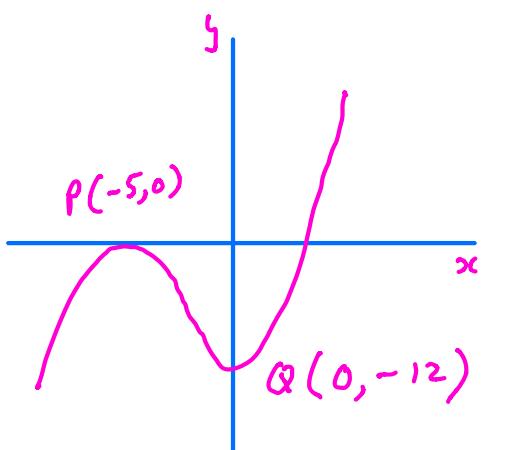
The points $P(-3, 0)$ and $Q(2, -4)$ are stationary points on the graph.

Sketch, on separate diagrams, the graphs of

(a) $y = 3f(x+2)$ (3)

(b) $y = |f(x)|$ (3)

On each diagram, show the coordinates of any stationary points.



7. The function f is defined by

$$f : x \mapsto \frac{3(x+1)}{2x^2 + 7x - 4} - \frac{1}{x+4}, \quad x \in \mathbb{R}, x > \frac{1}{2}$$

(a) Show that $f(x) = \frac{1}{2x-1}$ (4)

(b) Find $f^{-1}(x)$ (3)

(c) Find the domain of f^{-1} (1)

$$g(x) = \ln(x+1)$$

(d) Find the solution of $fg(x) = \frac{1}{7}$, giving your answer in terms of e. (4)

a) $f(x) = \frac{3(x+1)}{2x^2 + 7x - 4} - \frac{1}{x+4}$

$$f(x) = \frac{3(x+1)}{(2x-1)(x+4)} - \frac{1}{(x+4)}$$

$$f(x) = \frac{3(x+1) - 1(2x-1)}{(2x-1)(x+4)}$$

$$f(x) = \frac{3x+3 - 2x+1}{(2x-1)(x+4)}$$

$$f(x) = \frac{(x+4)}{(2x-1)(x+4)} = \frac{1}{2x-1}$$

b) Let $y = \frac{1}{2x-1}$

swap variables

$$x = \frac{1}{2y-1}$$



Question 7 continued

$$x(2y-1) = 1$$

$$2xy - x = 1$$

$$2xy = 1 + x$$

$$y = \frac{1+x}{2x}$$

$$f^{-1}(x) = \frac{1+x}{2x}$$

c) Domain of f^{-1} $x \in \mathbb{R}, x > 0$

d) $f(x) = \frac{1}{2x-1}$ $g(x) = \ln(x+1)$

$$fg(x) = f(\ln(x+1)) = \frac{1}{2\ln(x+1)-1}$$

Solve $\frac{1}{2\ln(x+1)-1} = \frac{1}{7}$

$$\Rightarrow 2\ln(x+1)-1 = 7$$

$$2\ln(x+1) = 8$$

$$\ln(x+1) = 4$$

$$x+1 = e^4$$

$$x = e^4 - 1$$



4.

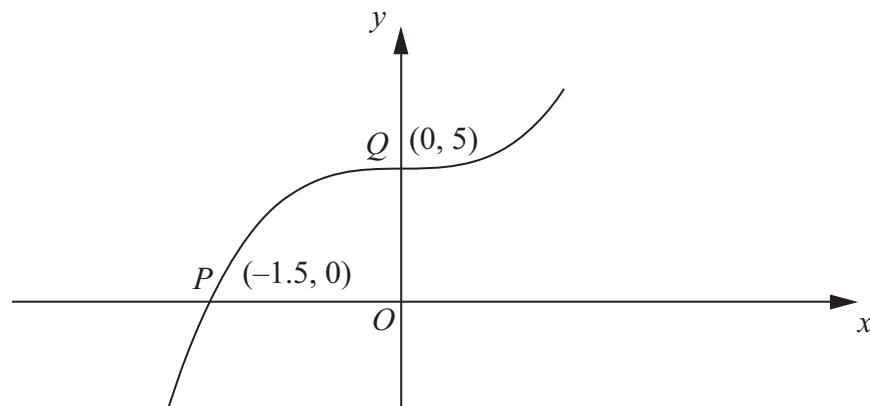
**Figure 2**

Figure 2 shows part of the curve with equation $y = f(x)$
The curve passes through the points $P(-1.5, 0)$ and $Q(0, 5)$ as shown.

On separate diagrams, sketch the curve with equation

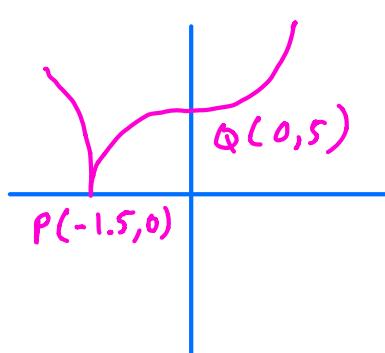
(a) $y = |f(x)|$ (2)

(b) $y = f(|x|)$ (2)

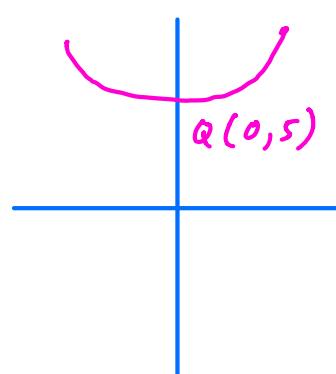
(c) $y = 2f(3x)$ (3)

Indicate clearly on each sketch the coordinates of the points at which the curve crosses or meets the axes.

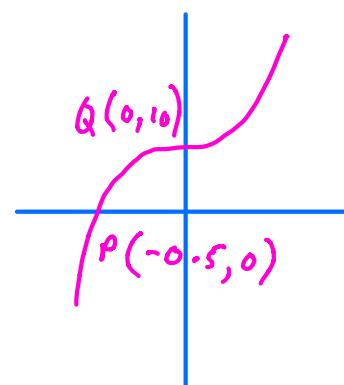
a) $y = |f(x)|$



b) $y = f(|x|)$



c) $y = 2f(3x)$



6. The functions f and g are defined by

$$f : x \mapsto e^x + 2, \quad x \in \mathbb{R}$$

$$g : x \mapsto \ln x, \quad x > 0$$

- (a) State the range of f . (1)
- (b) Find $fg(x)$, giving your answer in its simplest form. (2)
- (c) Find the exact value of x for which $f(2x+3)=6$ (4)
- (d) Find f^{-1} , the inverse function of f , stating its domain. (3)
- (e) On the same axes sketch the curves with equation $y = f(x)$ and $y = f^{-1}(x)$, giving the coordinates of all the points where the curves cross the axes. (4)

a) Range of f $f(x) \in \mathbb{R}, f(x) > 2$

b) $fg(x) = f(\ln x) = e^{\ln x} + 2 = x + 2$

c) $f(2x+3) = 6$

$$e^{2x+3} + 2 = 6$$

$$e^{2x+3} = 4$$

$$2x+3 = \ln 4$$

$$2x = \ln 4 - 3$$

$$x = \frac{\ln 4 - 3}{2}$$



Question 6 continued

d)

$$f(x) = e^x + 2$$

$$\text{Let } y = e^x + 2$$

swap variables

$$x = e^y + 2$$

$$x - 2 = e^y$$

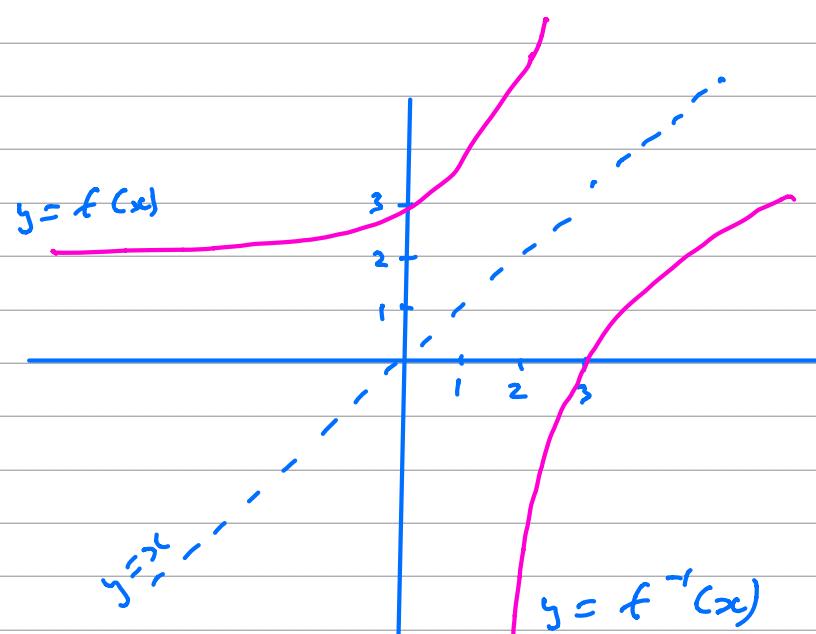
$$\ln(x-2) = y$$

$$f^{-1}(x) = \ln(x-2)$$

Domain of f^{-1}

$$x \in \mathbb{R}, x > 2$$

e)



3.

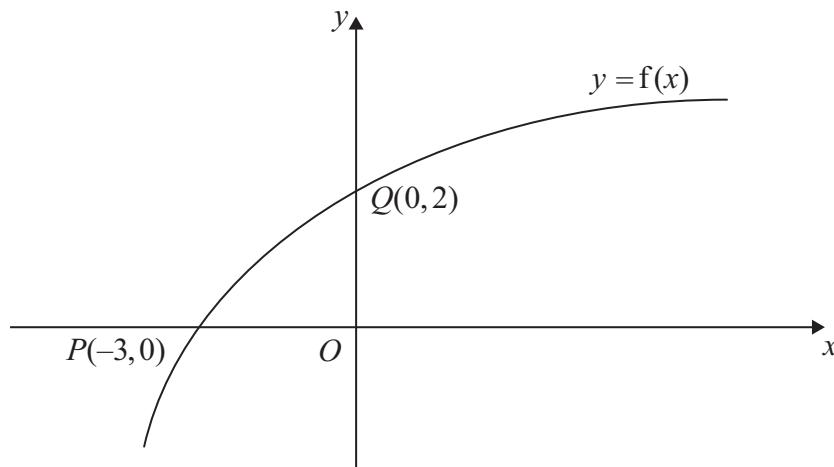
**Figure 1**

Figure 1 shows part of the curve with equation $y = f(x)$, $x \in \mathbb{R}$.

The curve passes through the points $Q(0, 2)$ and $P(-3, 0)$ as shown.

(a) Find the value of $f(f(-3))$. $\text{a) } f(f(-3))$
 $= f(0) = 2$ (2)

On separate diagrams, sketch the curve with equation

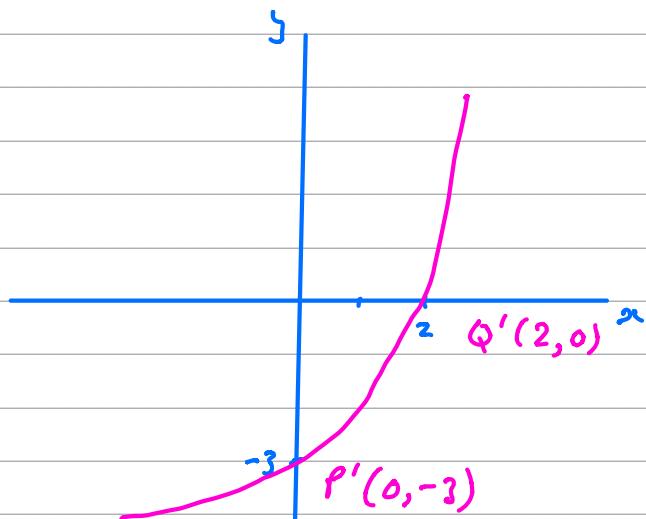
(b) $y = f^{-1}(x)$, $(\text{Mirror image in } y=x)$ (2)

(c) $y = f(|x|) - 2$, (2)

(d) $y = 2f\left(\frac{1}{2}x\right)$. (3)

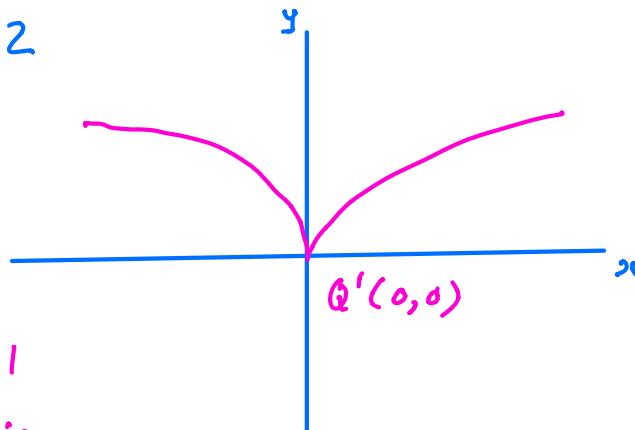
Indicate clearly on each sketch the coordinates of the points at which the curve crosses or meets the axes.

b) $y = f^{-1}(x)$



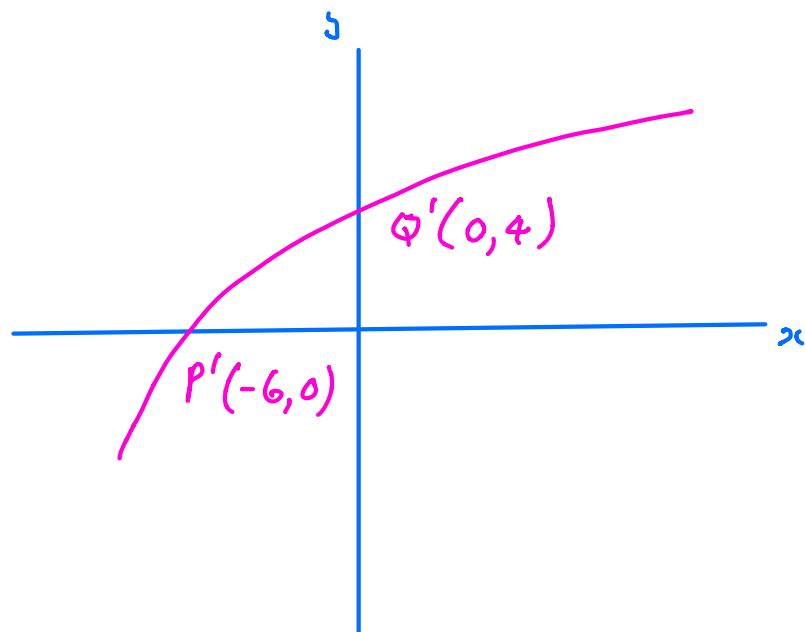
Question 3 continued

c) $y = f(|x|) - 2$



symmetrical
about y-axis

d) $y = 2f\left(\frac{x}{2}\right)$



P 4 1 4 8 6 A 0 7 2 8

2. Given that

$$f(x) = \ln x, \quad x > 0$$

sketch on separate axes the graphs of

(i) $y = f(x)$,

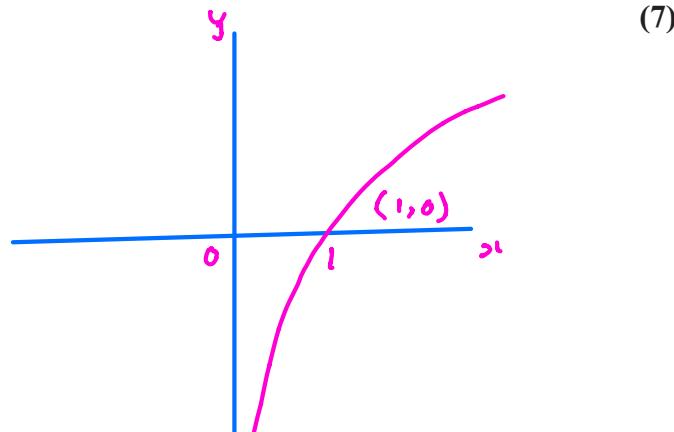
(ii) $y = |f(x)|$,

(iii) $y = -f(x-4)$.

Show, on each diagram, the point where the graph meets or crosses the x -axis.

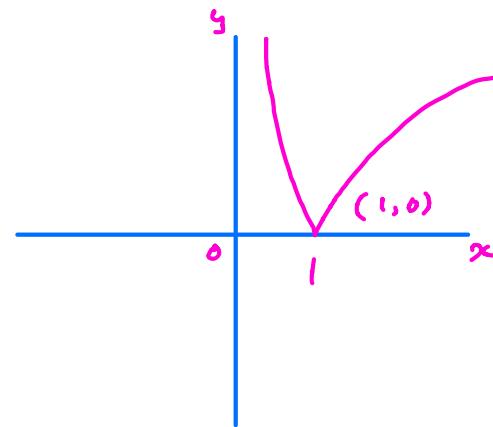
In each case, state the equation of the asymptote.

i) $y = f(x)$



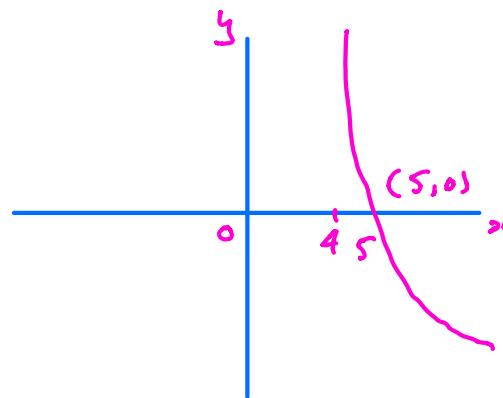
Asymptote $x = 0$

ii) $y = |f(x)|$



Asymptote $x = 0$

iii) $y = -f(x-4)$



Asymptote $x = 4$



7. The function f has domain $-2 \leq x \leq 6$ and is linear from $(-2, 10)$ to $(2, 0)$ and from $(2, 0)$ to $(6, 4)$. A sketch of the graph of $y = f(x)$ is shown in Figure 1.

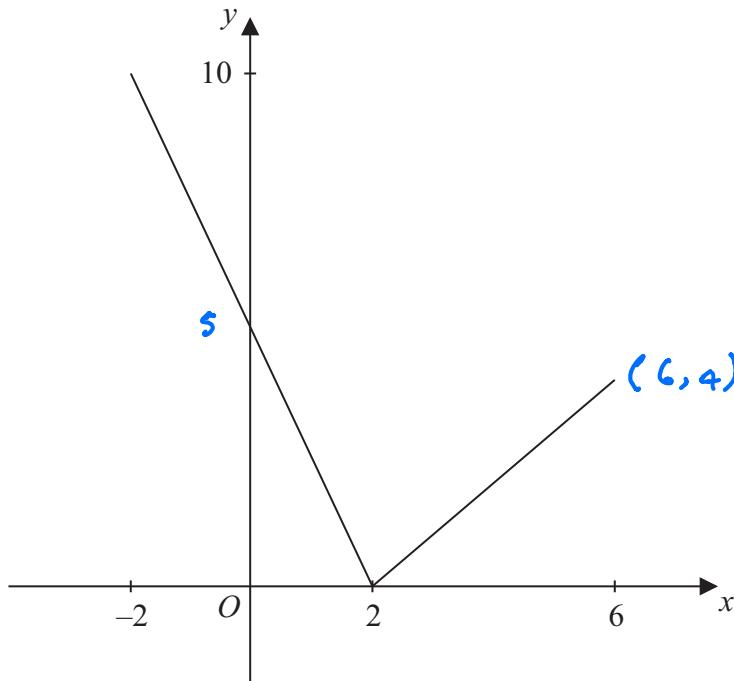


Figure 1

- (a) Write down the range of f .

$$\text{Range } 0 \leq f(x) \leq 10 \quad (1)$$

- (b) Find $ff(0)$.

$$ff(0) = f(5) = 3$$

(2)

The function g is defined by

$$g : x \rightarrow \frac{4 + 3x}{5 - x}, \quad x \in \mathbb{R}, \quad x \neq 5$$

- (c) Find $g^{-1}(x)$

(3)

- (d) Solve the equation $gf(x) = 16$

(5)

c)

$$\text{Let } y = \frac{4 + 3x}{5 - x}$$

Swap variables

$$x = \frac{4 + 3y}{5 - y}$$

$$x(5 - y) = 4 + 3y$$



Question 7 continued

$$5x - xy = 4 + 3y$$

$$5x - 4 = 3y + xy$$

$$5x - 4 = y(3+x)$$

$$\frac{5x-4}{3+x} = y$$

$$g^{-1}(x) = \frac{5x-4}{3+x}$$

d) Solve $gf(x) = 16$

$$f(x) = g^{-1}(16)$$

$$f(x) = \frac{5(16)-4}{3+16} = \frac{76}{19} = 4$$

$$f(x) = 4$$

From graph one solution is $x = 6$

Left section of graph is $y = -2.5x + 5$

Solve $-2.5x + 5 = 4$

$$5-4 = 2.5x$$

$$1 = \frac{5}{2}x$$

$$\frac{2}{5} = x$$

Solution $x = \frac{2}{5}$, $x = 6$

