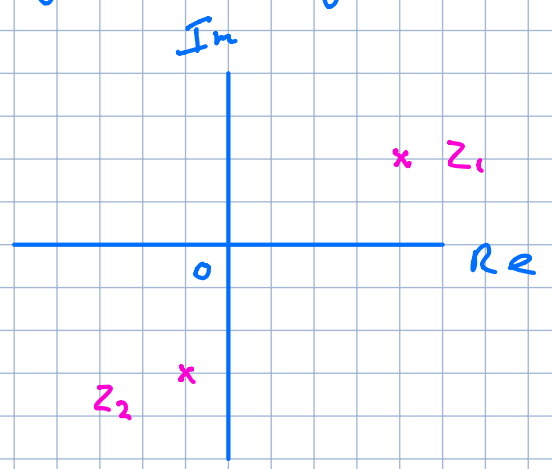
Argand Diagrams

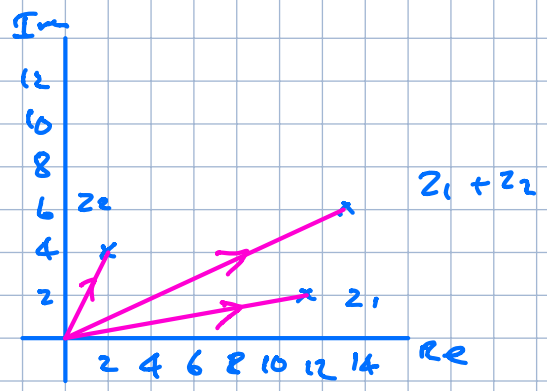


$z_1 = 4 + 2i$

$z_2 = -1 - 3i$

Exercise 2A

2)

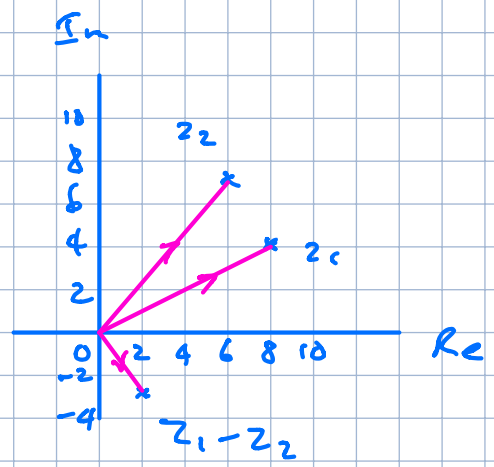


$z_1 = 11 + 2i$

$z_2 = 2 + 4i$

$z_1 + z_2 = 13 + 6i$

4)



$z_1 = 8 + 4i$

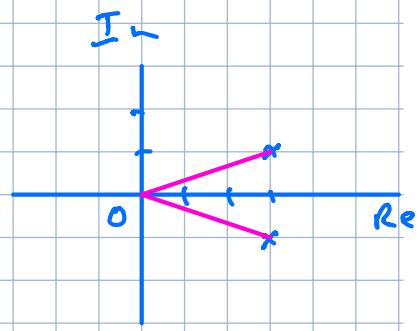
$z_2 = 6 + 7i$

$z_1 - z_2 = 2 - 3i$

8)

$z^2 - 6z + 10 = 0$

$z = \dfrac{6 \pm \sqrt{36 - 40}}{2} = 3 \pm i$



Modulus and Argument

Radian Measure

Degrees	0°	30°	45°	60°	90°	120°	135°	150°	180°
Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π

Principal angle θ given by $-\pi < \theta \leq \pi$

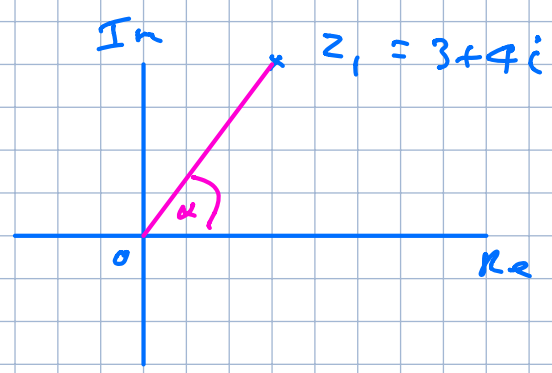
The modulus of a complex number $z = x + iy$ written as $|z|$ or $|x+iy|$ is the distance z is from the origin on an Argand diagram

$$|z| = \sqrt{x^2 + y^2}$$

The argument of $z = x + iy$ is the angle a line from O to z makes with the positive real axis.

This angle α is measured in radians

$$\alpha = \tan^{-1}\left(\frac{y}{x}\right)$$



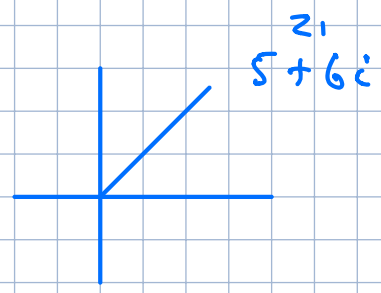
$$|z| = \sqrt{3^2 + 4^2} = 5$$

$$\arg(z) = \tan^{-1}\frac{4}{3}$$

$$= 0.927 \text{ radians}$$

Finding Moduli and Arguments

1) $z_1 = 5 + 6i$



$$|z_1| = \sqrt{5^2 + 6^2} = \sqrt{61}$$

$$\arg(z_1) = \tan^{-1}\frac{6}{5} = 0.876 \text{ radians}$$

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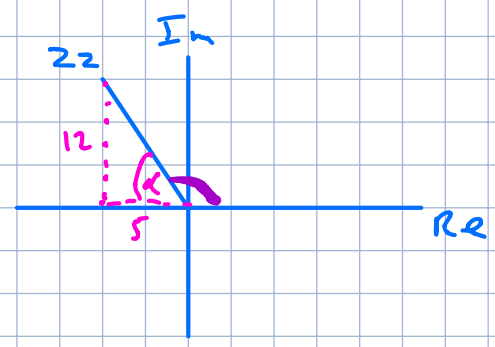
2) $z_2 = -5 + 12i$

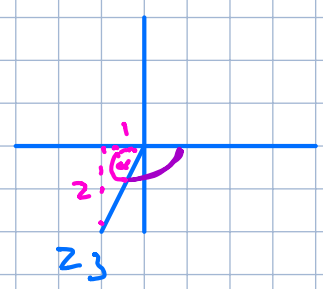


$\alpha = \tan^{-1}\frac{12}{5}$

$$|z_2| = \sqrt{5^2 + 12^2} = 13$$

$$\arg(z_2) = \pi - \tan^{-1}\frac{12}{5} = 1.97 \text{ radians}$$

3) $z_3 = -1 - 2i$



$\alpha = \tan^{-1}\frac{2}{1}$

$$|z_3| = \sqrt{(-1)^2 + (-2)^2} = \sqrt{5}$$

$$\arg(z_3) = -\pi + \alpha$$
$$= -\pi + \tan^{-1}\frac{2}{1} = -2.03 \text{ radians}$$

Exercise 2B

5) $z_1 = 4 + 6i \qquad z_2 = 1 + i$

a) $$\frac{z_1}{z_2} = \frac{4+6i}{1+i} = \frac{4+6i}{1+i} \times \frac{1-i}{1-i}$$

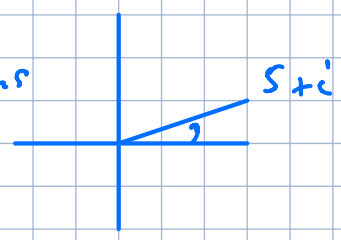
$$= \frac{4+6i-4i+6}{1^2+1^2}$$

$$= \frac{10+2i}{2}$$

$$= 5+i$$

$$\left|\frac{z_1}{z_2}\right| = \sqrt{5^2+1^2} = \sqrt{26}$$

$$\arg\left(\frac{z_1}{z_2}\right) = \tan^{-1}\frac{1}{5} = 0.197 \text{ radians}$$



$$|z_1| = \sqrt{4^2+6^2} = \sqrt{52}$$

$$|z_2| = \sqrt{1^2+1^2} = \sqrt{2}$$

$$\frac{\sqrt{52}}{\sqrt{2}} = \sqrt{26}$$

Exercise and Homework

Exercise 2B odd numbers 3 onwards