Midpoints

The midpoint of (x_1, y_1) and (x_2, y_2)

$$= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Examples

(x_1, y_1)	(x_2, y_2)	Midpoint
$(4, 6)$	$(8, 14)$	$\left(\frac{4+8}{2}, \frac{6+14}{2}\right) = (6, 10)$
$(5, 8)$	$(11, 14)$	$\left(\frac{5+11}{2}, \frac{8+14}{2}\right) = (8, 11)$
$(3, 7)$	$(10, 12)$	$\left(\frac{3+10}{2}, \frac{7+12}{2}\right) = \left(\frac{13}{2}, \frac{19}{2}\right)$
$(-3, 4)$	$(-1, -6)$	$\left(\frac{-3+-1}{2} \quad \frac{4+-6}{2}\right) = \left(\frac{-3-1}{2}, \frac{4-6}{2}\right) = (-2, -1)$

Exercise Find the midpoints

1) $(8, 11)$ and $(16, 17)$ $= \left(\frac{8+16}{2}, \frac{11+17}{2}\right) = (12, 14)$

2) $(3, 10)$ and $(8, 16)$ $= \left(\frac{3+8}{2}, \frac{10+16}{2}\right) = \left(\frac{11}{2}, 13\right)$

3) $(5, -1)$ and $(6, 11)$ $= \left(\frac{5+6}{2}, \frac{-1+11}{2}\right) = \left(\frac{11}{2}, 5\right)$

4) $(-2, 3)$ and $(-8, 5)$ $= \left(\frac{-2+-8}{2}, \frac{3+5}{2}\right) = (-5, 4)$

5) $(-7, -3)$ and $(11, 3)$ $= \left(\frac{-7+11}{2} \quad \frac{-3+3}{2}\right) = (2, 0)$

6) $(-5, -1)$ and $(-8, -7)$ $= \left(\frac{-5+-8}{2}, \frac{-1+-7}{2}\right) = \left(-\frac{13}{2}, -4\right)$

7) $(5,5)$ and $(7,9) = \left(\frac{5+7}{2}, \frac{5+9}{2}\right) = (6,7)$

8) $(10,-6)$ and $(3,1) = \left(\frac{10+3}{2}, \frac{-6+1}{2}\right) = \left(\frac{13}{2}, -\frac{5}{2}\right)$

9) $(4,-4)$ and $(-4,4) = \left(\frac{4+-4}{2}, \frac{-4+4}{2}\right) = (0,0)$

10) $(6,-4)$ and $(-2,-8) = \left(\frac{6+-2}{2}, \frac{-4+-8}{2}\right) = (2,-6)$

Equation of Line Through Two Given Points

Steps

1. First find gradient of line between the points
2. Write $y = mx + c$ where m is the gradient you have found
3. Substitute either of the two points to determine c

Example $(4,7)$ and $(6,13)$

$$m = \left(\frac{13-7}{6-4}\right) = \frac{6}{2} = 3$$

$$y = 3x + c$$

Sub $(4,7)$

$$7 = 3(4) + c$$
$$7 = 12 + c$$
$$7 - 12 = c$$
$$-5 = c$$

$$\underline{y = 3x - 5}$$

Example 2 $\overset{x_1\ y_1}{(-2,6)}$ and $\overset{x_2\ y_2}{(4,-6)}$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6-6}{4--2} = \frac{-12}{6} = -2$$

$$y = -2x + c$$

Sub $(-2, 6)$

$$6 = -2(-2) + c$$
$$6 = +4 + c$$
$$6 - 4 = c$$
$$2 = c$$

$$\underline{y = -2x + 2}$$

<u>Exercise</u> Find eqns of lines passing through:

1) $(4, 9)$ and $(7, 12)$

$$m = \frac{12-9}{7-4} = \frac{3}{3} = 1$$

$$y = x + c$$

Sub $(4, 9)$

$$9 = 4 + c$$
$$9 - 4 = c$$
$$5 = c$$

$$\underline{y = x + 5}$$

2) $(10, 7)$ and $(16, 4)$

$$m = \frac{4-7}{16-10} = \frac{-3}{6} = -\frac{1}{2}$$

$$y = -\frac{1}{2}x + c$$

Sub $(10, 7)$

$$7 = -\frac{1}{2}(10) + c$$
$$7 = -5 + c$$

$$c = 7 + 5 = 12$$

$$\underline{y = -\frac{1}{2}x + 12}$$

3) $(-1, 3)$ and $(3, 9)$

$$m = \frac{9-3}{3--1} = \frac{6}{4} = \frac{3}{2}$$

$$y = \frac{3}{2}x + c$$

Sub $(3, 9)$

$$9 = \frac{3}{2}(3) + c$$
$$9 = \frac{9}{2} + c$$
$$9 - \frac{9}{2} = c \qquad c = \frac{9}{2}$$

$$\underline{y = \frac{3}{2}x + \frac{9}{2}}$$

Find the Eqn of the perpendicular bisector between two points

Ex: $(2, 7)$ and $(4, 11)$

Midpoint $= \left(\frac{2+4}{2}, \frac{7+11}{2}\right) = (3, 9)$

gradient $m = \frac{11-7}{4-2} = \frac{4}{2} = 2$

perpendicular gradient $= -\frac{1}{2}$

Line is of form $y = -\frac{1}{2}x + c$

Sub $(3, 9)$

$$9 = -\frac{1}{2}(3) + c$$

$$9 = -\frac{3}{2} + c$$

$$9 + \frac{3}{2} = c$$

$$\frac{21}{2} = c$$

$$y = -\frac{1}{2}x + \frac{21}{2}$$