(i) $x = 2t$, $y = t^2$

(ii) $x = \cos 2\theta$, $y = \sin^2\theta$

(iii) $x = t^2$, $y = t^3$

(iv) $x = \sin^2\theta$, $y = 1 + 2\sin\theta$

(v) $x = 2\operatorname{cosec}\theta$, $y = 2\cot\theta$

(vi) $x = 2\sin^2\theta$, $y = 3\cos\theta$

(vii) $x = \tan\theta$, $y = \tan 2\theta$

(viii) $x = t^2$, $y = t^2 - t$

(ix) $x = \dfrac{t}{1+t}$, $y = \dfrac{t}{1-t}$

vi)

$$x = 2\sin^2\theta = 2(1-\cos^2\theta)$$

$$y = 3\cos\theta$$

$$\frac{y}{3} = \cos\theta$$

$$x = 2\left(1 - \frac{y^2}{9}\right)$$

$$x = 2 - \frac{2y^2}{9}$$

$$9x = 18 - 2y^2$$

ix)

$$x = \frac{t}{1+t} \qquad y = \frac{t}{1-t}$$

$$(1+t)x = t$$

$$x + tx = t$$

$$x = t - tx$$

$$x = t(1-x)$$

$$\frac{x}{1-x} = t$$

Sub for t

$$y = \frac{\frac{x}{1-x}}{1 - \frac{x}{1-x}}$$

$$y = \frac{\frac{x}{1-x}}{1-x-x}$$

$$\frac{}{1-x}$$

$$y = \frac{x}{1-2x}$$

---

vii) $x = \tan\theta \qquad y = \tan 2\theta$

$$y = \frac{\tan\theta + \tan\theta}{1 - \tan^2\theta}$$

$$y = \frac{2x}{1-x^2}$$

Exercise 9B

1 i) $x = 3t^2 \qquad y = 2t^3$

$$\frac{dx}{dt} = 6t \qquad \frac{dy}{dt} = 6t^2$$

$$\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt} = \frac{6t^2}{6t} = t$$

---

1 ii) $x = \theta - \cos\theta \qquad y = \theta + \sin\theta$

$$\frac{dx}{d\theta} = 1 + \sin\theta \qquad \frac{dy}{d\theta} = 1 + \cos\theta$$

$$\frac{dy}{dx} = \frac{dy}{d\theta} \Big/ \frac{dx}{d\theta} = \frac{1 + \cos\theta}{1 + \sin\theta}$$

1 vii) $x = e^{2t} + 1$ $\qquad$ $y = e^{t}$

$$\frac{dx}{dt} = 2e^{2t} \qquad \frac{dy}{dt} = e^{t}$$

$$\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt} = \frac{e^{t}}{2e^{2t}} = \frac{1}{2e^{t}}$$

---

3) $x = t^2$ $\qquad$ $y = 1 - \frac{1}{2t}$ $\qquad$ $t > 0$

i) cuts x-axis when $y = 0$

$$\Rightarrow 1 - \frac{1}{2t} = 0$$

$$\Rightarrow t = \frac{1}{2}$$

$$x = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

Cuts x-axis at $P\left(\frac{1}{4}, 0\right)$

---

ii) $x = t^2$ $\qquad$ $y = 1 - \frac{1}{2t}$

$$\frac{dx}{dt} = 2t \qquad y = 1 - \frac{1}{2}t^{-1}$$

$$\frac{dy}{dt} = \frac{1}{2}t^{-2} = \frac{1}{2t^2}$$

$$\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt} = \frac{\frac{1}{2t^2}}{2t} = \frac{1}{4t^3}$$

At P, $t=\frac{1}{2}$ $\quad \frac{dy}{dx} = \frac{1}{4(\frac{1}{2})^3} = \frac{1}{\frac{1}{2}} = 2$

---

iii) tgt $\quad y - y_1 = m(x - x_1)$

$P(\frac{1}{4}, 0)$ $\quad y - 0 = 2(x - \frac{1}{4})$

$y = 2x - \frac{1}{2}$

---

iv) Cuts y-axis when $x = 0$

$y = 0 - \frac{1}{2}$

$y = -\frac{1}{2}$

Cuts y-axis at $(0, -\frac{1}{2})$

---

5) $x = \cos\theta \qquad y = \cos 2\theta$

i) $\frac{dx}{d\theta} = -\sin\theta \qquad \frac{dy}{d\theta} = -2\sin 2\theta$

$$\frac{dy}{dx} = \frac{dy}{d\theta} \Big/ \frac{dx}{d\theta} = \frac{-2\sin 2\theta}{-\sin\theta}$$

$$= -\frac{4\sin\theta\cos\theta}{-\sin\theta} = 4\cos\theta$$

---

ii) $\frac{dy}{dx} = 4\cos\theta = 4x$

$\frac{d^2y}{dx^2} = 4$

$\Rightarrow \frac{d^2y}{dx^2} - 4 = 4 - 4 = 0$

---

9) $x = 1 - 2t \qquad y = t^2$

i) $x = 5$

$\Rightarrow 5 = 1 - 2t$

$2t = 1 - 5$

$2t = -4$

$t = -2$

$t = -2$

$\Rightarrow y = (-2)^2$

$y = 4$

$P(5, 4)$ on curve when $t = -2$

---

ii) $\frac{dx}{dt} = -2 \qquad \frac{dy}{dt} = 2t$

$\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt} = \frac{2t}{-2} = -t$

---

iii) At P $t = -2$ $\therefore \frac{dy}{dx} = --2 = 2$

$y - y_1 = m(x - x_1)$

$P(5, 4)$

$$y - 4 = 2(x - 5)$$

$$y - 4 = 2x - 10$$

$$y = 2x - 6$$

Normal has gradient $-\frac{1}{2}$

$$y - y_1 = m(x - x_1)$$

$$y - 4 = -\frac{1}{2}(x - 5)$$

$$y - 4 = -\frac{1}{2}x + \frac{5}{2}$$

$$y = -\frac{1}{2}x + \frac{13}{2} \qquad *$$

Find point on curve that is also on normal

$y = t^2, \quad x = 1 - 2t$

Sub in $*$

$$t^2 = -\frac{1}{2}(1 - 2t) + \frac{13}{2}$$

$$2t^2 = -1(1 - 2t) + 13$$

$$2t^2 = -1 + 2t + 13$$

$$2t^2 - 2t - 12 = 0$$

$$t^2 - t - 6 = 0$$

$$(t - 3)(t + 2) = 0$$

$$t = 3 \quad \text{or} \quad t = -2$$

$$P$$

$t = -3 \Rightarrow x = 1 - 2(3) = -5, \quad y = 3^2 = 9$

$\therefore Q(-5, 9)$

Classwork and Homework

Q2, Q4, Q8, Q10, Q12

Hand in Q8, Q10, Q12 tomorrow
