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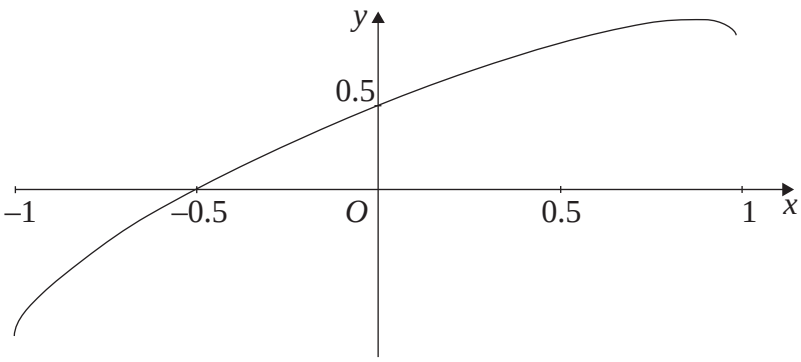
4. **Figure 2**



The curve shown in Figure 2 has parametric equations

$$x = \sin t, \quad y = \sin\left(t + \tfrac{\pi}{6}\right), \qquad -\tfrac{\pi}{2} < t < \tfrac{\pi}{2}.$$

(a) Find an equation of the tangent to the curve at the point where $t = \frac{\pi}{6}$.

(6)

(b) Show that a cartesian equation of the curve is

$$y = \frac{\sqrt{3}}{2}x + \frac{1}{2}\sqrt{(1 - x^2)}, \qquad -1 < x < 1.$$

(3)

Mark Scheme 2 Pages Further On

N 2 3 5 6 3 A 0 8 2 0

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Question 4 continued

Q4

(Total 9 marks)

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Question Number	Scheme		Marks
4. (a)	$x = \sin t, \quad y = \sin\left(t + \frac{\pi}{6}\right)$		
	$\frac{dx}{dt} = \cos t, \quad \frac{dy}{dt} = \cos\left(t + \frac{\pi}{6}\right)$	Attempt to differentiate both x and y wrt t to give two terms in cos Correct $\frac{dx}{dt}$ and $\frac{dy}{dt}$	M1 A1
	When $t = \frac{\pi}{6}$, $\frac{dy}{dx} = \frac{\cos\left(\frac{\pi}{6} + \frac{\pi}{6}\right)}{\cos\left(\frac{\pi}{6}\right)} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \text{awrt } 0.58$	Divides in correct way and substitutes for t to give any of the four underlined oe: Ignore the double negative if candidate has differentiated $\sin \to -\cos$	A1
	When $t = \frac{\pi}{6}, \quad x = \frac{1}{2}, \; y = \frac{\sqrt{3}}{2}$	The point $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ or $\left(\frac{1}{2}, \text{awrt } 0.87\right)$	B1
	T: $y - \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(x - \frac{1}{2}\right)$	Finding an equation of a tangent with their point and their tangent gradient or finds c and uses $y = (\text{their gradient})x + \text{"c"}$. Correct EXACT equation of tangent oe.	dM1 A1 oe
	or $\frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(\frac{1}{2}\right) + c \;\Rightarrow\; c = \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{6} = \frac{\sqrt{3}}{3}$		
	or **T**: $\left[y = \frac{\sqrt{3}}{3}x + \frac{\sqrt{3}}{3}\right]$		**[6]**
(b)	$y = \sin\left(t + \frac{\pi}{6}\right) = \sin t \cos\frac{\pi}{6} + \cos t \sin\frac{\pi}{6}$	Use of compound angle formula for sine.	M1
	Nb: $\sin^2 t + \cos^2 t \equiv 1 \;\Rightarrow\; \cos^2 t \equiv 1 - \sin^2 t$		
	$\therefore x = \sin t$ gives $\cos t = \sqrt{\left(1 - x^2\right)}$	Use of trig identity to find $\cos t$ in terms of x or $\cos^2 t$ in terms of x.	M1
	$\therefore y = \frac{\sqrt{3}}{2}\sin t + \frac{1}{2}\cos t$		
	gives $y = \frac{\sqrt{3}}{2}x + \frac{1}{2}\sqrt{\left(1 - x^2\right)}$ **AG**	Substitutes for $\sin t$, $\cos\frac{\pi}{6}$, $\cos t$ and $\sin\frac{\pi}{6}$ to give y in terms of x.	A1 cso **[3]**
			9 marks

edexcel

Question Number	Scheme		Marks
Aliter **4.** (a) **Way 2**	$x = \sin t, \quad y = \sin\left(t + \frac{\pi}{6}\right) = \sin t \cos\frac{\pi}{6} + \cos t \sin\frac{\pi}{6}$	(Do not give this for part (b)) Attempt to differentiate x and y wrt t to give $\frac{dx}{dt}$ in terms of cos and $\frac{dy}{dt}$ in the form $\pm a\cos t \pm b\sin t$	M1
	$\frac{dx}{dt} = \cos t, \quad \frac{dy}{dt} = \cos t \cos\frac{\pi}{6} - \sin t \sin\frac{\pi}{6}$	Correct $\frac{dx}{dt}$ and $\frac{dy}{dt}$	A1
	When $t = \frac{\pi}{6}$, $\frac{dy}{dx} = \underline{\frac{\cos\frac{\pi}{6}\cos\frac{\pi}{6} - \sin\frac{\pi}{6}\sin\frac{\pi}{6}}{\cos\left(\frac{\pi}{6}\right)}}$ $= \underline{\frac{\frac{3}{4} - \frac{1}{4}}{\frac{\sqrt{3}}{2}} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \text{awrt } 0.58}$	Divides in correct way and substitutes for t to give any of the four underlined oe:	A1
	When $t = \frac{\pi}{6}$, $x = \frac{1}{2}$, $y = \frac{\sqrt{3}}{2}$	The point $\underline{\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)}$ or $\underline{\left(\frac{1}{2}, \text{awrt } 0.87\right)}$	B1
	T: $\underline{y - \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(x - \frac{1}{2}\right)}$	Finding an equation of a tangent with their point and their tangent gradient or finds c and uses $y = (\text{their gradient})x + "c"$.	dM1
	or $\frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(\frac{1}{2}\right) + c \Rightarrow c = \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{6} = \frac{\sqrt{3}}{3}$	Correct EXACT equation of <u>tangent</u> oe.	<u>A1</u> oe
	or **T**: $\left[\underline{y = \frac{\sqrt{3}}{3}x + \frac{\sqrt{3}}{3}}\right]$		**[6]**

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Question Number	Scheme		Marks
Aliter **4.** (a) **Way 3**	$y = \frac{\sqrt{3}}{2}x + \frac{1}{2}\sqrt{(1-x^2)}$		
	$\frac{dy}{dx} = \frac{\sqrt{3}}{2} + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)(1-x^2)^{-\frac{1}{2}}(-2x)$	Attempt to differentiate two terms using the chain rule for the second term. Correct $\frac{dy}{dx}$	M1 A1
	$\frac{dy}{dx} = \frac{\sqrt{3}}{2} + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)(1-(0.5)^2)^{-\frac{1}{2}}(-2(0.5)) = \frac{1}{\sqrt{3}}$	Correct substitution of $x = \frac{1}{2}$ into a correct $\frac{dy}{dx}$	A1
	When $t = \frac{\pi}{6}$, $x = \frac{1}{2}$, $y = \frac{\sqrt{3}}{2}$	The point $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ or $\left(\frac{1}{2}, \text{awrt } 0.87\right)$	B1
	T: $y - \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(x - \frac{1}{2}\right)$	Finding an equation of a tangent with their point and their tangent gradient or finds c and uses $y = (\text{their gradient})x + "c"$. Correct EXACT equation of tangent oe.	dM1 A1 oe
	or $\frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(\frac{1}{2}\right) + c \Rightarrow c = \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{6} = \frac{\sqrt{3}}{3}$		
	or **T**: $\left[y = \frac{\sqrt{3}}{3}x + \frac{\sqrt{3}}{3}\right]$		**[6]**
Aliter **4.** (b) **Way 2**	$x = \sin t$ gives $y = \frac{\sqrt{3}}{2}\sin t + \frac{1}{2}\sqrt{(1-\sin^2 t)}$	Substitutes $x = \sin t$ into the equation give in y.	M1
	Nb: $\sin^2 t + \cos^2 t \equiv 1 \Rightarrow \cos^2 t \equiv 1 - \sin^2 t$		
	$\cos t = \sqrt{(1-\sin^2 t)}$	Use of trig identity to deduce that $\cos t = \sqrt{(1-\sin^2 t)}$.	M1
	gives $y = \frac{\sqrt{3}}{2}\sin t + \frac{1}{2}\cos t$		
	Hence $y = \sin t \cos\frac{\pi}{6} + \cos t \sin\frac{\pi}{6} = \sin\left(t + \frac{\pi}{6}\right)$	Using the compound angle formula to prove $y = \sin\left(t + \frac{\pi}{6}\right)$	A1 cso **[3]**
			9 marks