| $\begin{array}{ll} \text { 8(i) } & \text { At } \mathrm{P}, x \cos 3 x=0 \\ \Rightarrow & \cos 3 x=0 \\ \Rightarrow & 3 x=\pi / 2,3 \pi / 2 \\ \Rightarrow & x=\pi / 6, \pi / 2 \\ & \text { So } \mathrm{P} \text { is }(\pi / 6,0) \text { and } \mathrm{Q} \text { is }(\pi / 2,0) \end{array}$ | M1 <br> M1 <br> A1 A1 <br> [4] | or verification $3 x=\pi / 2,(3 \pi / 2 \ldots)$ <br> dep both Ms condone degrees here |
| :---: | :---: | :---: |
| $\begin{aligned} & \text { (ii) } \frac{d y}{d x}=-3 x \sin 3 x+\cos 3 x \\ & \qquad \text { At } \mathrm{P}, \frac{d y}{d x}=-\frac{\pi}{2} \sin \frac{\pi}{2}+\cos \frac{\pi}{2}=-\frac{\pi}{2} \\ & \text { At TPs } \frac{d y}{d x}=-3 x \sin 3 x+\cos 3 x=0 \\ & \Rightarrow \quad \cos 3 x=3 x \sin 3 x \\ & \Rightarrow \quad 1=3 x \sin 3 x / \cos 3 x=3 x \tan 3 x \\ & \Rightarrow \quad x \tan 3 x=1 / 3 * \end{aligned}$ | M1 <br> B1 <br> A1 <br> M1 <br> A1cao <br> M1 <br> E1 <br> [7] | Product rule $\mathrm{d} / \mathrm{d} x(\cos 3 x)=-3 \sin 3 x$ cao (so for $\mathrm{d} y / \mathrm{d} x=-3 x \sin 3 x$ allow B1) mark final answer substituting their $-\pi / 6$ (must be rads) $-\pi / 2$ <br> $\mathrm{d} y / \mathrm{d} x=0$ and $\sin 3 x / \cos 3 x=\tan 3 x$ used <br> www |
| $\text { (iii) } \begin{aligned} & A=\int_{0}^{\pi / 6} x \cos 3 x d x \\ & \text { Parts with } u=x, \mathrm{~d} v / \mathrm{d} x=\cos 3 x \\ & \mathrm{~d} u / \mathrm{d} x=1, v=1 / 3 \sin 3 x \\ & \Rightarrow \quad A=\left[\frac{1}{3} x \sin 3 x\right]_{0}^{\frac{\pi}{6}}-\int_{0}^{\pi / 6} \frac{1}{3} \sin 3 x d x \\ &=\left[\frac{1}{3} x \sin 3 x+\frac{1}{9} \cos 3 x\right]_{0}^{\frac{\pi}{6}} \\ &=\frac{\pi}{18}-\frac{1}{9} \end{aligned}$ | B1 <br> M1 <br> A1 <br> A1 <br> M1dep <br> A1 cao <br> [6] | Correct integral and limits (soi) - ft their P , but must be in radians <br> can be without limits <br> dep previous A1. <br> substituting correct limits, dep $1^{\text {st }} \mathrm{M} 1$ : ft their P provided in radians <br> o.e. but must be exact |


| $\begin{aligned} & \text { 9(i) } \quad \mathrm{f}^{\prime}(x)=\frac{\left(x^{2}+1\right) 4 x-\left(2 x^{2}-1\right) 2 x}{\left(x^{2}+1\right)^{2}} \\ &=\frac{4 x^{3}+4 x-4 x^{3}+2 x}{\left(x^{2}+1\right)^{2}}=\frac{6 x}{\left(x^{2}+1\right)^{2}} * \\ & \Rightarrow \quad \text { When } x>0,6 x>0 \text { and }\left(x^{2}+1\right)^{2}>0 \\ & \Rightarrow \quad \mathrm{f}^{\prime}(x)>0 \end{aligned}$ | M1 <br> A1 <br> E1 <br> M1 <br> E1 <br> [5] | Quotient or product rule correct expression www <br> attempt to show or solve $\mathrm{f}^{\prime}(x)>0$ <br> numerator $>0$ and denominator $>0$ or, if solving, $6 x>0 \Rightarrow x>0$ |
| :---: | :---: | :---: |
| $\begin{aligned} & \text { (ii) } \mathrm{f}(2)=\frac{8-1}{4+1}=1 \frac{2}{5} \\ & \text { Range is }-1 \leq y \leq 1 \frac{2}{5} \end{aligned}$ | B1 <br> B1 <br> [2] | must be $\leq, y$ or $\mathrm{f}(x)$ |
| $\begin{array}{ll} \text { (iii) } & \mathrm{f}^{\prime}(x) \max \text { when } \mathrm{f}^{\prime \prime}(x)=0 \\ \Rightarrow & 6-18 x^{2}=0 \\ \Rightarrow & x^{2}=1 / 3, x=1 / \sqrt{ } 3 \\ \Rightarrow & \mathrm{f}^{\prime}(x)=\frac{6 / \sqrt{3}}{\left(1 \frac{1}{3}\right)^{2}}=\frac{6}{\sqrt{3}} \cdot \frac{9}{16}=\frac{9 \sqrt{3}}{8}=1.95 \end{array}$ | $\begin{gathered} \text { M1 } \\ \text { A1 } \\ \text { M1 } \\ \text { A1 } \\ {[4]} \end{gathered}$ | $( \pm) 1 / \sqrt{3}$ oe $(0.577$ or better $)$ substituting $1 / \sqrt{3}$ into $\mathrm{f}^{\prime}(x)$ $9 \sqrt{ } 3 / 8$ o.e. or 1.95 or better (1.948557..) |
| (iv) Domain is $-1<x<1 \frac{2}{5}$ <br> Range is $0 \leq y \leq 2$ | B1 <br> B1 <br> M1 <br> A1 cao <br> [4] | ft their 1.4 but not $x \geq-1$ <br> or $0 \leq \mathrm{g}(x) \leq 2($ not f$)$ <br> Reasonable reflection in $y=x$ <br> from $(-1,0)$ to $(1.4,2)$, through $(0, \sqrt{ } 2 / 2)$ <br> allow omission of one of $-1,1.4,2, \sqrt{ } 2 / 2$ |
| $\begin{array}{ll} \text { (v) } & y=\frac{2 x^{2}-1}{x^{2}+1} \quad x \leftrightarrow y \\ & x=\frac{2 y^{2}-1}{y^{2}+1} \\ \Rightarrow & x y^{2}+x=2 y^{2}-1 \\ \Rightarrow & x+1=2 y^{2}-x y^{2}=y^{2}(2-x) \\ \Rightarrow & y^{2}=\frac{x+1}{2-x} \\ \Rightarrow & y=\sqrt{\frac{x+1}{2-x}} * \end{array}$ | M1 <br> M1 <br> M1 <br> E1 <br> [4] | (could start from g) <br> Attempt to invert <br> clearing fractions collecting terms in $y^{2}$ and factorising <br> www |

