

Section B (36 marks)

- 8** Fig. 8 shows the line $y = x$ and parts of the curves $y = f(x)$ and $y = g(x)$, where

$$f(x) = e^{x-1}, \quad g(x) = 1 + \ln x.$$

The curves intersect the axes at the points A and B, as shown. The curves and the line $y = x$ meet at the point C.

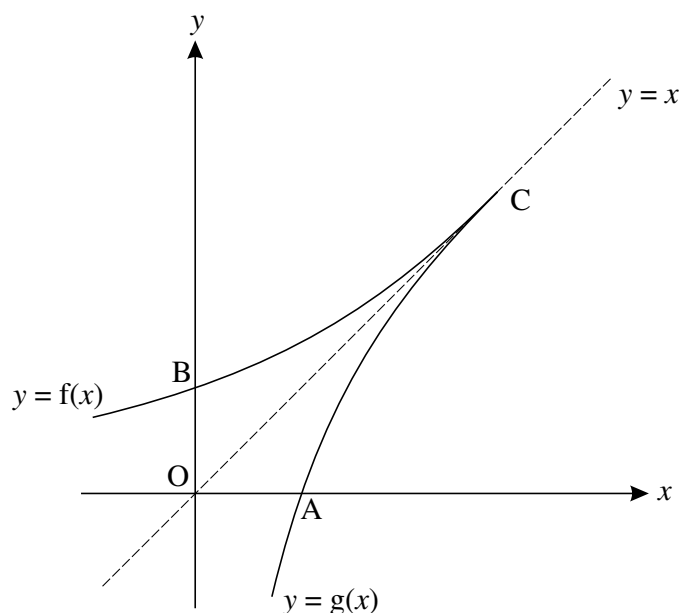


Fig. 8

- (i) Find the exact coordinates of A and B. Verify that the coordinates of C are (1, 1). [5]
- (ii) Prove algebraically that $g(x)$ is the inverse of $f(x)$. [2]
- (iii) Evaluate $\int_0^1 f(x) dx$, giving your answer in terms of e . [3]
- (iv) Use integration by parts to find $\int \ln x dx$.
Hence show that $\int_{e^{-1}}^1 g(x) dx = \frac{1}{e}$. [6]
- (v) Find the area of the region enclosed by the lines OA and OB, and the arcs AC and BC. [2]

- 9 Fig. 9 shows the curve $y = \frac{x^2}{3x-1}$.

P is a turning point, and the curve has a vertical asymptote $x = a$.

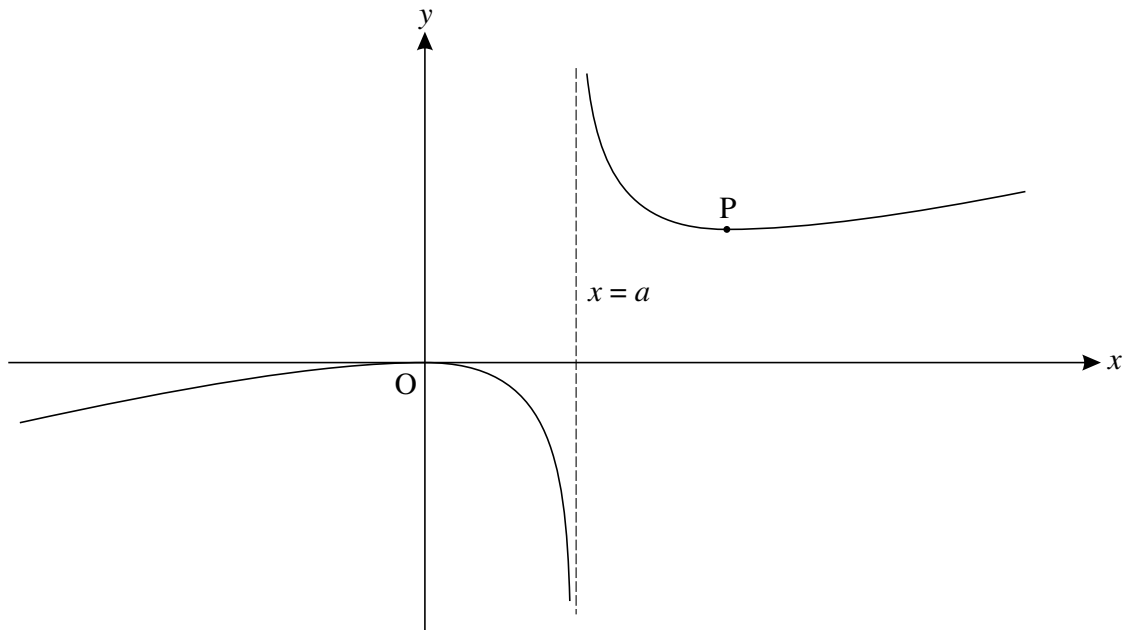


Fig. 9

- (i) Write down the value of a . [1]
- (ii) Show that $\frac{dy}{dx} = \frac{x(3x-2)}{(3x-1)^2}$. [3]
- (iii) Find the exact coordinates of the turning point P.

Calculate the gradient of the curve when $x = 0.6$ and $x = 0.8$, and hence verify that P is a minimum point. [7]

- (iv) Using the substitution $u = 3x - 1$, show that $\int \frac{x^2}{3x-1} dx = \frac{1}{27} \int \left(u + 2 + \frac{1}{u} \right) du$.

Hence find the exact area of the region enclosed by the curve, the x -axis and the lines $x = \frac{2}{3}$ and $x = 1$. [7]

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