Section B (36 marks)

8 Fig. 8 shows the line $y = x$ and parts of the curves $y = f(x)$ and $y = g(x)$, where

$$f(x) = e^{x-1}, \quad g(x) = 1 + \ln x.$$

The curves intersect the axes at the points A and B, as shown. The curves and the line $y = x$ meet at the point C.

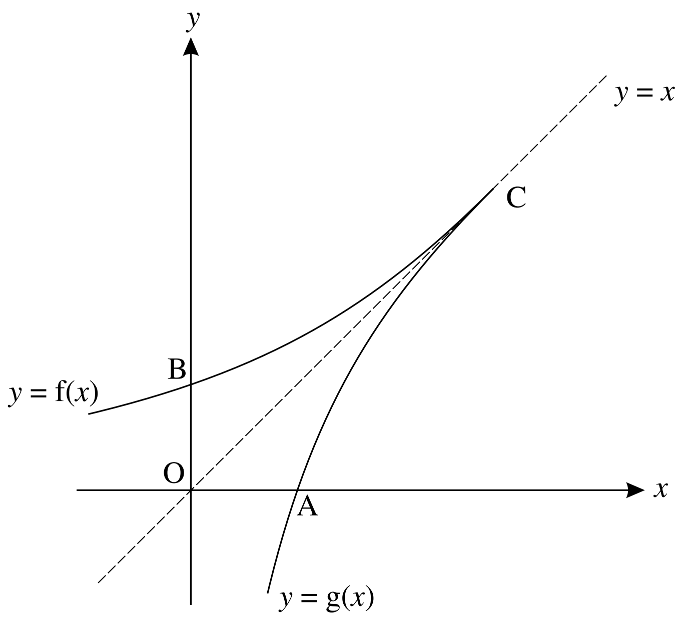

Fig. 8

(i) Find the exact coordinates of A and B. Verify that the coordinates of C are (1, 1). **[5]**

(ii) Prove algebraically that $g(x)$ is the inverse of $f(x)$. **[2]**

(iii) Evaluate $\int_0^1 f(x)\,dx$, giving your answer in terms of e. **[3]**

(iv) Use integration by parts to find $\int \ln x\,dx$.

Hence show that $\int_{e^{-1}}^{1} g(x)\,dx = \frac{1}{e}$. **[6]**

(v) Find the area of the region enclosed by the lines OA and OB, and the arcs AC and BC. **[2]**

9 Fig. 9 shows the curve $y = \dfrac{x^2}{3x - 1}$.

P is a turning point, and the curve has a vertical asymptote $x = a$.

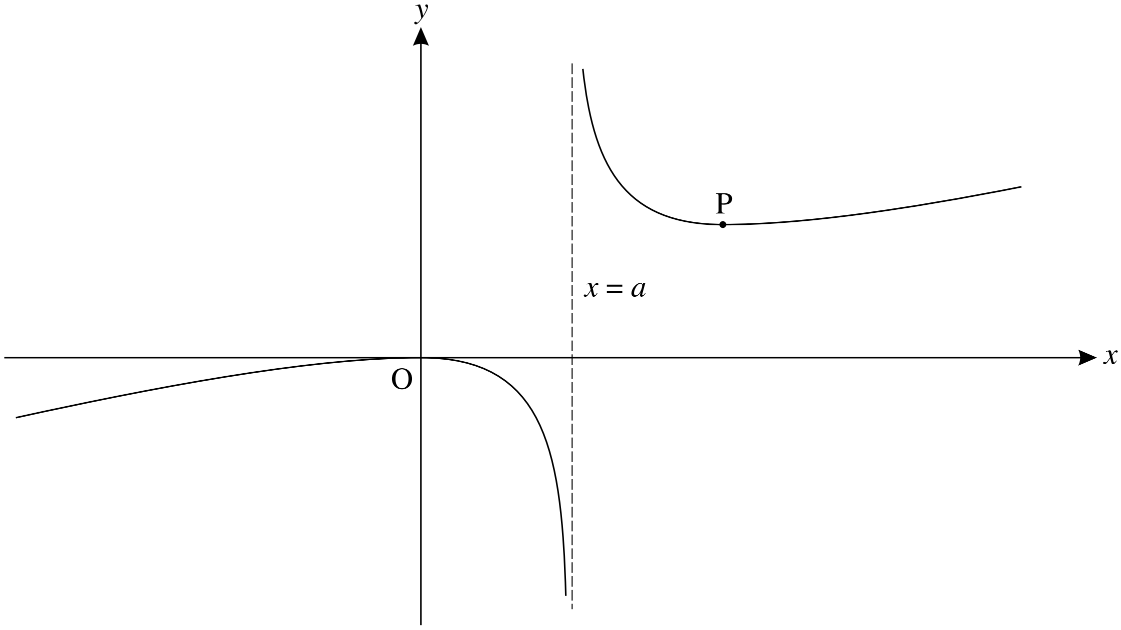

Fig. 9

(i) Write down the value of a. **[1]**

(ii) Show that $\dfrac{dy}{dx} = \dfrac{x(3x - 2)}{(3x - 1)^2}$. **[3]**

(iii) Find the exact coordinates of the turning point P.

Calculate the gradient of the curve when $x = 0.6$ and $x = 0.8$, and hence verify that P is a minimum point. **[7]**

(iv) Using the substitution $u = 3x - 1$, show that $\displaystyle\int \frac{x^2}{3x - 1}\,dx = \frac{1}{27}\int \left(u + 2 + \frac{1}{u}\right) du$.

Hence find the exact area of the region enclosed by the curve, the x-axis and the lines $x = \frac{2}{3}$ and $x = 1$. **[7]**