8(i) A: $1 + \ln x = 0$ $\Rightarrow$ $\ln x = -1$ so A is $(e^{-1}, 0)$ $\Rightarrow$ $x = e^{-1}$ B: $x = 0, y = e^{0-1} = e^{-1}$ so B is $(0, e^{-1})$ C: $f(1) = e^{1-1} = e^0 = 1$ $g(1) = 1 + \ln 1 = 1$	M1 A1 B1 E1 E1 [5]	 SC1 if obtained using symmetry condone use of symmetry Penalise A = e^{-1}, B = e^{-1}, or co-ords wrong way round, but condone labelling errors.
(ii) *Either* by invertion: e.g. $y = e^{x-1}$ $x \leftrightarrow y$ $x = e^{y-1}$ $\Rightarrow$ $\ln x = y - 1$ $\Rightarrow$ $1 + \ln x = y$	 M1 E1	 taking lns or exps
or by composing e.g. $f\,g(x) = f(1 + \ln x)$ $= e^{1 + \ln x - 1}$ $= e^{\ln x} = x$	 M1 E1 [2]	 $e^{1 + \ln x - 1}$ or $1 + \ln(e^{x-1})$
(iii) $\int_0^1 e^{x-1}dx = \left[e^{x-1}\right]_0^1$ $= e^0 - e^{-1}$ $= 1 - e^{-1}$	M1 M1 A1cao [3]	$\left[e^{x-1}\right]$ o.e or $u = x - 1 \Rightarrow \left[e^u\right]$ substituting correct limits for x or u o.e. not e^0, must be exact.
(iv) $\int \ln x dx = \int \ln x \frac{d}{dx}(x)dx$ $= x\ln x - \int x.\frac{1}{x}dx$ $= x \ln x - x + c$ $\Rightarrow$ $\int_{e^{-1}}^1 g(x)dx = \int_{e^{-1}}^1 (1+\ln x)dx$ $= \left[x + x\ln x - x\right]_{e^{-1}}^1$ $= \left[x\ln x)\right]_{e^{-1}}^1$ $= 1\ln 1 - e^{-1}\ln(e^{-1})$ $= e^{-1}$ *	M1 A1 A1cao B1ft DM1 E1 [6]	parts: $u = \ln x$, $du/dx = 1/x$, $v = x$, $dv/dx = 1$ condone no 'c' ft their '$x \ln x - x$' (provided 'algebraic') substituting limits dep B1 www
(v) Area = $\int_0^1 f(x)\,dx - \int_{e^{-1}}^1 g(x)\,dx$ $= (1 - e^{-1}) - e^{-1}$ $= 1 - 2/e$	M1 A1cao	Must have correct limits 0.264 or better.
or Area OCB = area under curve – triangle $= 1 - e^{-1} - \frac{1}{2} \times 1 \times 1$ $= \frac{1}{2} - e^{-1}$ *or* Area OAC = triangle – area under curve $= \frac{1}{2} \times 1 \times 1 - e^{-1}$ $= \frac{1}{2} - e^{-1}$ Total area = $2(\frac{1}{2} - e^{-1}) = 1 - 2/e$	 M1 A1cao [2]	 OCA or OCB = $\frac{1}{2} - e^{-1}$ 0.264 or better

9(i) $a = 1/3$	B1 [1]	or 0.33 or better
(ii) $\frac{dy}{dx} = \frac{(3x-1)2x - x^2.3}{(3x-1)^2}$ $= \frac{6x^2 - 2x - 3x^2}{(3x-1)^2}$ $= \frac{3x^2 - 2x}{(3x-1)^2}$ $= \frac{x(3x-2)}{(3x-1)^2}$ *	M1 A1 E1 [3]	quotient rule www – must show both steps; penalise missing brackets.
(iii) dy/dx = 0 when $x(3x-2) = 0$ $\Rightarrow$ $x = 0$ or $x = 2/3$, so at P, $x = 2/3$ when $x = \frac{2}{3}$, $y = \frac{(2/3)^2}{3\times(2/3)-1} = \frac{4}{9}$ when $x = 0.6$, dy/dx = –0.1875 when $x = 0.8$, dy/dx = 0.1633 Gradient increasing $\Rightarrow$ minimum	M1 A1 M1 A1cao B1 B1 E1 [7]	if denom = 0 also then M0 o.e e.g. $0.\dot{6}$, but must be exact o.e e.g. $0.\dot{4}$, but must be exact –3/16, or –0.19 or better 8/49 or 0.16 or better o.e. e.g. 'from negative to positive'. Allow ft on their gradients, provided –ve and +ve respectively. Accept table with indications of signs of gradient.
(iv) $\int \frac{x^2}{3x-1} dx \quad u = 3x-1 \Rightarrow du = 3dx$ $= \int \frac{\frac{(u+1)^2}{9}}{u} \frac{1}{3} du$ $= \frac{1}{27}\int \frac{(u+1)^2}{u} du = \frac{1}{27}\int \frac{u^2+2u+1}{u} du$ $= \frac{1}{27}\int (u + 2 + \frac{1}{u}) du$ * Area = $\int_{2/3}^{1} \frac{x^2}{3x-1} dx$ When $x = 2/3$, $u = 1$, when $x = 1$, $u = 2$ $= \frac{1}{27}\int_1^2 (u + 2 + 1/u) du$ $= \frac{1}{27}\left[\frac{1}{2}u^2 + 2u + \ln u\right]_1^2$ $= \frac{1}{27}[(2 + 4 + \ln 2) - (\frac{1}{2} + 2 + \ln 1)]$ $= \frac{1}{27}(3\frac{1}{2} + \ln 2)$ $[= \frac{7 + 2\ln 2}{54}]$	B1 M1 M1 E1 B1 M1 A1cao [7]	$\frac{\frac{(u+1)^2}{9}}{u}$ o.e. × 1/3 (du) expanding Condone missing du's $\left[\frac{1}{2}u^2 + 2u + \ln u\right]$ substituting correct limits, dep integration o.e., but must evaluate ln 1 = 0 and collect terms.