8(i) When $x = 1$ $y = 1^2 - (\ln 1)/8 = 1$ Gradient of PR = $(1 + 7/8)/1 = 1\frac{7}{8}$	B1 M1 A1 [3]	 1.9 or better
(ii) $\frac{dy}{dx} = 2x - \frac{1}{8x}$ When $x = 1$, $dy/dx = 2 - 1/8 = 1\frac{7}{8}$ Same as gradient of PR, so PR touches curve	B1 B1dep E1 [3]	cao 1.9 or better dep 1st B1 dep gradients exact
(iii) Turning points when $dy/dx = 0$ $\Rightarrow\quad 2x - \frac{1}{8x} = 0$ $\Rightarrow\quad 2x = \frac{1}{8x}$ $\Rightarrow\quad x^2 = 1/16$ $\Rightarrow\quad x = ¼\ (x > 0)$ When $x = ¼$, $y = \frac{1}{16} - \frac{1}{8}\ln\frac{1}{4} = \frac{1}{16} + \frac{1}{8}\ln 4$ So TP is $(\frac{1}{4}, \frac{1}{16} + \frac{1}{8}\ln 4)$	M1 M1 A1 M1 A1cao [5]	setting their derivative to zero multiplying through by x allow verification substituting for x in y o.e. but must be exact, not $1/4^2$. Mark final answer.
(iv) $\frac{d}{dx}(x\ln x - x) = x.\frac{1}{x} + 1.\ln x - 1 = \ln x$	M1 A1	product rule $\ln x$
Area = $\int_1^2 (x^2 - \frac{1}{8}\ln x)dx$ $= \left[\frac{1}{3}x^3 - \frac{1}{8}(x\ln x - x)\right]_1^2$ $= (\frac{8}{3} - \frac{1}{4}\ln 2 + \frac{1}{4}) - (\frac{1}{3} - \frac{1}{8}\ln 1 + \frac{1}{8})$ $= \frac{7}{3} + \frac{1}{8} - \frac{1}{4}\ln 2$ $= \frac{59}{24} - \frac{1}{4}\ln 2$ *	M1 M1 A1 M1 E1 [7]	correct integral and limits (soi) – condone no dx $\int \ln x dx = x\ln x - x$ used (or derived using integration by parts) $\frac{1}{3}x^3 - \frac{1}{8}(x\ln x - x)$ – bracket required substituting correct limits must show at least one step

9(i) Asymptotes when $(\sqrt{})(2x - x^2) = 0$ $\Rightarrow x(2 - x) = 0$ $\Rightarrow x = 0$ or 2 so $a = 2$ Domain is $0 < x < 2$	M1 A1 B1ft [3]	 or by verification $x > 0$ and $x < 2$, not $\leq$
(ii) $y = (2x - x^2)^{-1/2}$ let $u = 2x - x^2$, $y = u^{-1/2}$ $\Rightarrow$ $dy/du = -\frac{1}{2} u^{-3/2}$, $du/dx = 2 - 2x$ $\Rightarrow$ $\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx} = -\frac{1}{2}(2x - x^2)^{-3/2}.(2 - 2x)$ $= \frac{x - 1}{(2x - x^2)^{3/2}}$ *	M1 B1 A1 E1	chain rule (or within correct quotient rule) $-\frac{1}{2} u^{-3/2}$ or $-\frac{1}{2}(2x - x^2)^{-3/2}$ or $\frac{1}{2}(2x - x^2)^{-1/2}$ in quotient rule $\times (2 - 2x)$ www – penalise missing brackets here
$dy/dx = 0$ when $x - 1 = 0$ $\Rightarrow$ $x = 1$, $y = 1/\sqrt{(2 - 1)} = 1$ Range is $y \geq 1$	M1 A1 B1 B1ft [8]	extraneous solutions M0
(iii) (*A*) $g(-x) = \frac{1}{\sqrt{1-(-x)^2}} = \frac{1}{\sqrt{1-x^2}} = g(x)$	M1 E1	Expression for $g(-x)$ – must have $g(-x) = g(x)$ seen
(*B*) $g(x-1) = \frac{1}{\sqrt{1-(x-1)^2}}$ $= \frac{1}{\sqrt{1 - x^2 + 2x - 1}} = \frac{1}{\sqrt{2x - x^2}} = f(x)$	M1 E1	 must expand bracket
(*C*) $f(x)$ is $g(x)$ translated 1 unit to the right. But $g(x)$ is symmetrical about O*y* So $f(x)$ is symmetrical about $x = 1$.	M1 M1 A1	 dep both M1s
or $f(1 - x) = g(-x)$, $f(1 + x) = g(x)$ $\Rightarrow$ $f(1 + x) = f(1 - x)$ $\Rightarrow$ $f(x)$ is symmetrical about $x = 1$.	M1 E1 A1 [7]	or $f(1-x) = \frac{1}{\sqrt{2-2x-(1-x)^2}} = \frac{1}{\sqrt{2-2x-1+2x-x^2}} = \frac{1}{\sqrt{1-x^2}}$ $f(1+x) = \frac{1}{\sqrt{2+2x-(1+x)^2}} = \frac{1}{\sqrt{2+2x-1-2x-x^2}} = \frac{1}{\sqrt{1-x^2}}$