Section B (36 marks)

7 A curve is defined by the equation $y = 2x \ln(1 + x)$.

(i) Find $\frac{dy}{dx}$ and hence verify that the origin is a stationary point of the curve. [4]

(ii) Find $\frac{d^2y}{dx^2}$, and use this to verify that the origin is a minimum point. [5]

(iii) Using the substitution $u = 1 + x$, show that $\int \frac{x^2}{1+x} \, dx = \int \left(u - 2 + \frac{1}{u}\right) du$.

Hence evaluate $\int_0^1 \frac{x^2}{1+x} \, dx$, giving your answer in an exact form. [6]

(iv) Using integration by parts and your answer to part **(iii)**, evaluate $\int_0^1 2x \ln(1 + x) \, dx$. [4]

8 Fig. 8 shows the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = 1 + \sin 2x$ for $-\frac{1}{4}\pi \leqslant x \leqslant \frac{1}{4}\pi$.

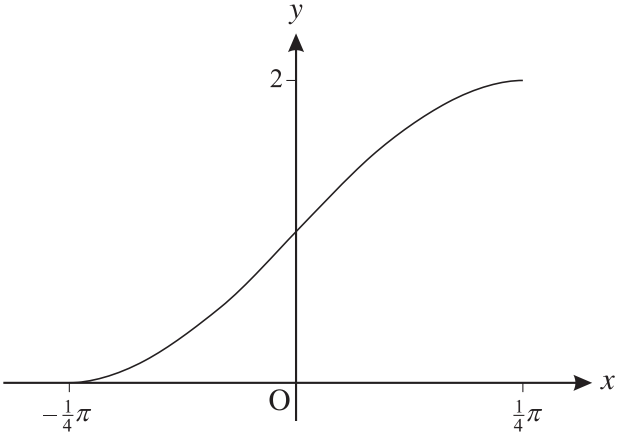

Fig. 8

(i) State a sequence of two transformations that would map part of the curve $y = \sin x$ onto the curve $y = \mathrm{f}(x)$. [4]

(ii) Find the area of the region enclosed by the curve $y = \mathrm{f}(x)$, the x-axis and the line $x = \frac{1}{4}\pi$. [4]

(iii) Find the gradient of the curve $y = \mathrm{f}(x)$ at the point $(0, 1)$. Hence write down the gradient of the curve $y = \mathrm{f}^{-1}(x)$ at the point $(1, 0)$. [4]

(iv) State the domain of $\mathrm{f}^{-1}(x)$. Add a sketch of $y = \mathrm{f}^{-1}(x)$ to a copy of Fig. 8. [3]

(v) Find an expression for $\mathrm{f}^{-1}(x)$. [2]