Section B (36 marks)

7 Fig. 7 shows part of the curve $y = f(x)$, where $f(x) = x\sqrt{1+x}$. The curve meets the x-axis at the origin and at the point P.

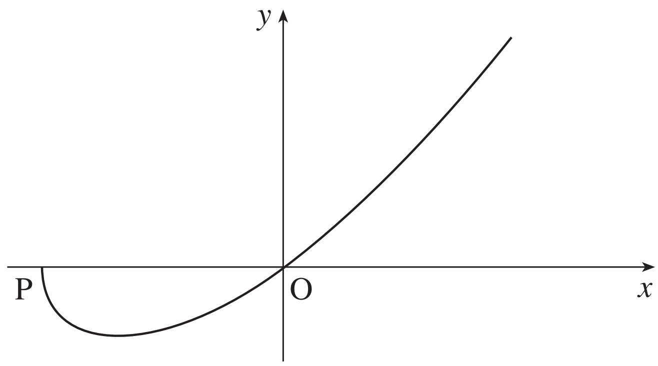

Fig. 7

(i) Verify that the point P has coordinates $(-1, 0)$. Hence state the domain of the function $f(x)$. [2]

(ii) Show that $\dfrac{dy}{dx} = \dfrac{2+3x}{2\sqrt{1+x}}$. [4]

(iii) Find the exact coordinates of the turning point of the curve. Hence write down the range of the function. [4]

(iv) Use the substitution $u = 1 + x$ to show that

$$\int_{-1}^{0} x\sqrt{1+x}\,dx = \int_{0}^{1} \left(u^{\frac{3}{2}} - u^{\frac{1}{2}}\right) du.$$

Hence find the area of the region enclosed by the curve and the x-axis. [8]

8 Fig. 8 shows part of the curve $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = (\mathrm{e}^x - 1)^2 \quad \text{for } x \geqslant 0.$$

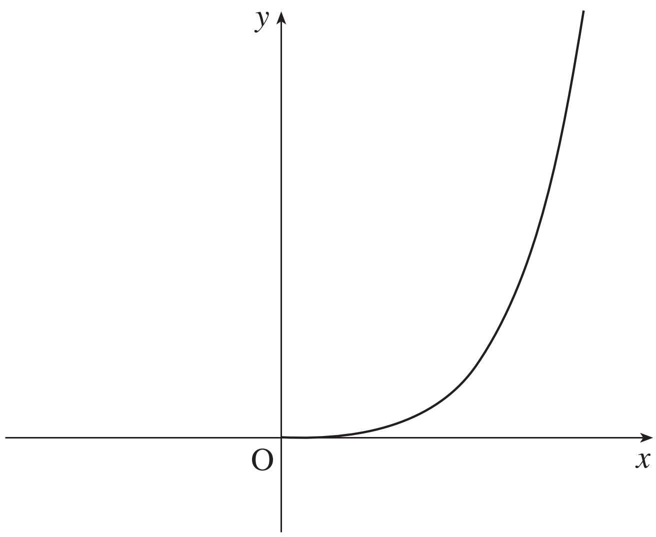

Fig. 8

(i) Find $\mathrm{f}'(x)$, and hence calculate the gradient of the curve $y = \mathrm{f}(x)$ at the origin and at the point $(\ln 2, 1)$. [5]

The function $\mathrm{g}(x)$ is defined by $\mathrm{g}(x) = \ln\left(1 + \sqrt{x}\right)$ for $x \geqslant 0$.

(ii) Show that $\mathrm{f}(x)$ and $\mathrm{g}(x)$ are inverse functions. Hence sketch the graph of $y = \mathrm{g}(x)$.

Write down the gradient of the curve $y = \mathrm{g}(x)$ at the point $(1, \ln 2)$. [5]

(iii) Show that $\int \left(\mathrm{e}^x - 1\right)^2 \mathrm{d}x = \frac{1}{2}\mathrm{e}^{2x} - 2\mathrm{e}^x + x + c.$

Hence evaluate $\int_0^{\ln 2} \left(\mathrm{e}^x - 1\right)^2 \mathrm{d}x$, giving your answer in an exact form. [5]

(iv) Using your answer to part **(iii)**, calculate the area of the region enclosed by the curve $y = \mathrm{g}(x)$, the x-axis and the line $x = 1$. [3]