

Section B

7(i) When $x = -1, y = -1\sqrt{0} = 0$ Domain $x \geq -1$	E1 B1 [2]	Not $y \geq -1$
(ii) $\frac{dy}{dx} = x \cdot \frac{1}{2}(1+x)^{-1/2} + (1+x)^{1/2}$ $= \frac{1}{2}(1+x)^{-1/2}[x + 2(1+x)]$ $= \frac{2+3x}{2\sqrt{1+x}} *$	B1 B1 M1 E1	$x \cdot \frac{1}{2}(1+x)^{-1/2}$ $\dots + (1+x)^{1/2}$ taking out common factor or common denominator www
$or u = x + 1 \Rightarrow du/dx = 1$ $\Rightarrow y = (u-1)u^{1/2} = u^{3/2} - u^{1/2}$ $\Rightarrow \frac{dy}{du} = \frac{3}{2}u^{\frac{1}{2}} - \frac{1}{2}u^{-\frac{1}{2}}$ $\Rightarrow \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{3}{2}(x+1)^{\frac{1}{2}} - \frac{1}{2}(x+1)^{-\frac{1}{2}}$ $= \frac{1}{2}(x+1)^{-\frac{1}{2}}(3x+3-1)$ $= \frac{2+3x}{2\sqrt{1+x}} *$	M1 A1 M1 E1 [4]	taking out common factor or common denominator
(iii) $dy/dx = 0$ when $3x+2=0$ $\Rightarrow x = -2/3$ $\Rightarrow y = -\frac{2}{3}\sqrt{\frac{1}{3}}$ Range is $y \geq -\frac{2}{3}\sqrt{\frac{1}{3}}$	M1 A1ca o A1 B1 ft [4]	 o.e. not $x \geq -\frac{2}{3}\sqrt{\frac{1}{3}}$ (ft their y value, even if approximate)
(iv) $\int_{-1}^0 x\sqrt{1+x} dx$ let $u = 1+x, du/dx = 1 \Rightarrow du = dx$ when $x = -1, u = 0, \text{ when } x = 0, u = 1$ $= \int_0^1 (u-1)\sqrt{u} du$ $= \int_0^1 (u^{3/2} - u^{1/2}) du *$	M1 B1 M1 E1	$du = dx$ or $du/dx = 1$ or $dx/du = 1$ changing limits – allow with no working shown provided limits are present and consistent with dx and du . $(u-1)\sqrt{u}$ www – condone only final brackets missing, otherwise notation must be correct
$= \left[\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2} \right]_0^1$ $= \pm \frac{4}{15}$	B1 B1 M1 A1ca o [8]	$\frac{2}{5}u^{5/2}, -\frac{2}{3}u^{3/2}$ (oe) substituting correct limits (can imply the zero limit) $\pm \frac{4}{15}$ or ± 0.27 or better, not 0.26

<p>8 (i) $f(x) = 2(e^x - 1)e^x$</p> <p>When $x = 0$, $f'(0) = 0$</p> <p>When $x = \ln 2$, $f'(\ln 2) = 2(2 - 1)2 = 4$</p>	M1 A1 B1dep M1 A1cao [5]	or $f(x) = e^{2x} - 2e^x + 1$ M1 (or $(e^x)^2 - 2e^x + 1$ plus correct deriv of $(e^x)^2$) $\Rightarrow f'(x) = 2e^{2x} - 2e^x$ A1 derivative must be correct, www $e^{\ln 2} = 2$ soi
<p>(ii) $y = (e^x - 1)^2$ $x \leftrightarrow y$</p> $x = (e^y - 1)^2$ $\Rightarrow \sqrt{x} = e^y - 1$ $\Rightarrow 1 + \sqrt{x} = e^y$ $\Rightarrow y = \ln(1 + \sqrt{x})$	M1 M1 E1	reasonable attempt to invert formula taking lns similar scheme of inverting $y = \ln(1 + \sqrt{x})$
<p>or $gf(x) = g((e^x - 1)^2)$ $= \ln(1 + e^x - 1)$ $= x$</p>	M1 M1 E1	constructing gf or fg $\ln(e^x) = x$ or $e^{\ln(1+\sqrt{x})} = 1 + \sqrt{x}$
<p>Gradient at $(1, \ln 2) = \frac{1}{4}$</p>	B1 B1ft [5]	reflection in $y = x$ (must have infinite gradient at origin)
<p>(iii) $\int (e^x - 1)^2 dx = \int (e^{2x} - 2e^x + 1) dx$</p> $= \frac{1}{2}e^{2x} - 2e^x + x + c *$ $\int_0^{\ln 2} (e^x - 1)^2 dx = \left[\frac{1}{2}e^{2x} - 2e^x + x \right]_0^{\ln 2}$ $= \frac{1}{2}e^{2\ln 2} - 2e^{\ln 2} + \ln 2 - (\frac{1}{2} - 2)$ $= 2 - 4 + \ln 2 - \frac{1}{2} + 2$ $= \ln 2 - \frac{1}{2}$	M1 E1 M1 M1 A1 [5]	expanding brackets (condone e^{x^2}) substituting limits $e^{\ln 2} = 2$ used must be exact
<p>(iv)</p> <p>Area = $1 \times \ln 2 - (\ln 2 - \frac{1}{2})$ $= \frac{1}{2}$</p>	M1 B1 A1cao [3]	subtracting area in (iii) from rectangle rectangle area = $1 \times \ln 2$ must be supported