Section B (36 marks)

7 Fig. 7 shows the curve $y = \dfrac{x^2 + 3}{x - 1}$. It has a minimum at the point P. The line l is an asymptote to the curve.

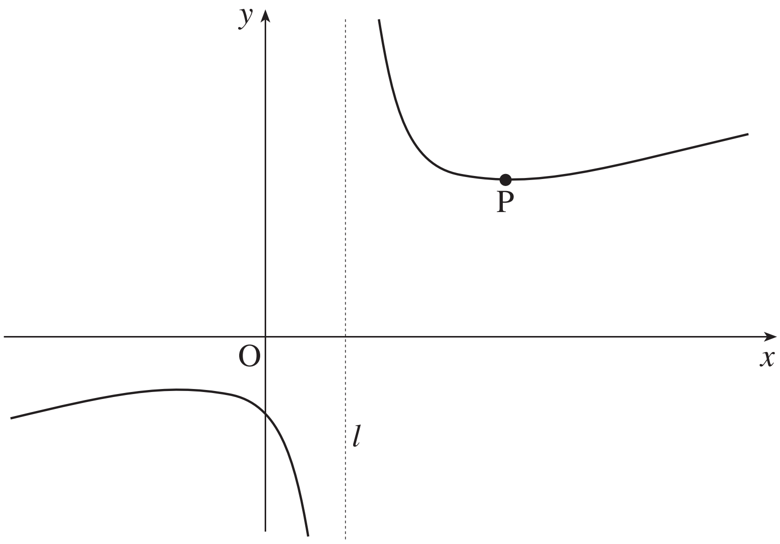

Fig. 7

(i) Write down the equation of the asymptote l. [1]

(ii) Find the coordinates of P. [6]

(iii) Using the substitution $u = x - 1$, show that the area of the region enclosed by the x-axis, the curve and the lines $x = 2$ and $x = 3$ is given by

$$\int_1^2 \left(u + 2 + \frac{4}{u} \right) \mathrm{d}u.$$

Evaluate this area exactly. [7]

(iv) Another curve is defined by the equation $\mathrm{e}^y = \dfrac{x^2 + 3}{x - 1}$. Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y by differentiating implicitly. Hence find the gradient of this curve at the point where $x = 2$. [4]

8 Fig. 8 shows part of the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \mathrm{e}^{-\frac{1}{5}x}\sin x$, for all x.

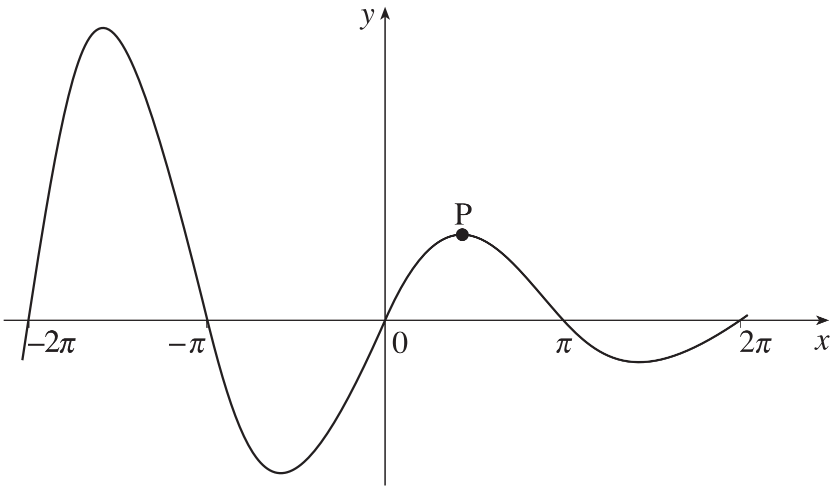

Fig. 8

(i) Sketch the graphs of

(A) $y = \mathrm{f}(2x)$,

(B) $y = \mathrm{f}(x + \pi)$. [4]

(ii) Show that the x-coordinate of the turning point P satisfies the equation $\tan x = 5$.

Hence find the coordinates of P. [6]

(iii) Show that $\mathrm{f}(x + \pi) = -\mathrm{e}^{-\frac{1}{5}\pi}\mathrm{f}(x)$. Hence, using the substitution $u = x - \pi$, show that

$$\int_{\pi}^{2\pi} \mathrm{f}(x)\,\mathrm{d}x = -\mathrm{e}^{-\frac{1}{5}\pi}\int_{0}^{\pi} \mathrm{f}(u)\,\mathrm{d}u.$$

Interpret this result graphically. [You should *not* attempt to integrate $\mathrm{f}(x)$.] [8]