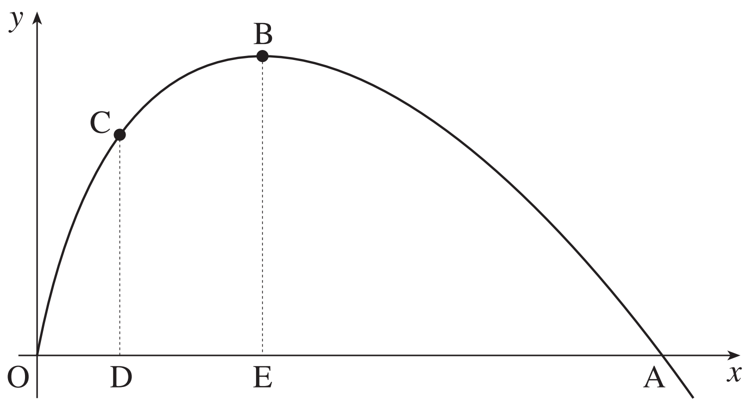
Section B (36 marks)

7 Fig. 7 shows the curve

$$y = 2x - x\ln x, \text{ where } x > 0.$$

The curve crosses the x-axis at A, and has a turning point at B. The point C on the curve has x-coordinate 1. Lines CD and BE are drawn parallel to the y-axis.



Not to scale

Fig. 7

(i) Find the x-coordinate of A, giving your answer in terms of e. [2]

(ii) Find the exact coordinates of B. [6]

(iii) Show that the tangents at A and C are perpendicular to each other. [3]

(iv) Using integration by parts, show that

$$\int x \ln x \, dx = \tfrac{1}{2}x^2 \ln x - \tfrac{1}{4}x^2 + c.$$

Hence find the exact area of the region enclosed by the curve, the x-axis and the lines CD and BE. [7]

[Question 8 is printed overleaf.]

8 The function $\mathrm{f}(x) = \dfrac{\sin x}{2 - \cos x}$ has domain $-\pi \leqslant x \leqslant \pi$.

Fig. 8 shows the graph of $y = \mathrm{f}(x)$ for $0 \leqslant x \leqslant \pi$.

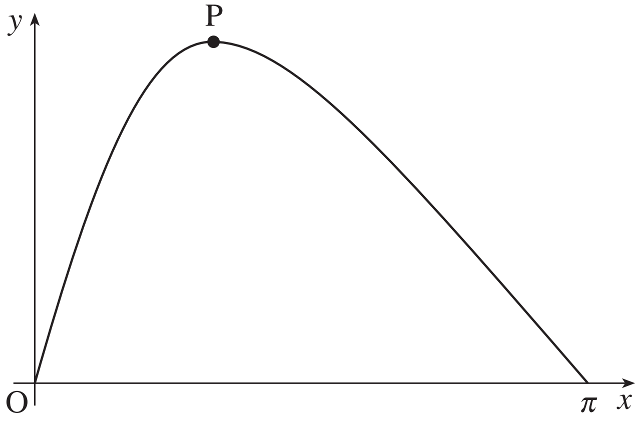


Fig. 8

(i) Find $\mathrm{f}(-x)$ in terms of $\mathrm{f}(x)$. Hence sketch the graph of $y = \mathrm{f}(x)$ for the complete domain $-\pi \leqslant x \leqslant \pi$. [3]

(ii) Show that $\mathrm{f}'(x) = \dfrac{2\cos x - 1}{(2 - \cos x)^2}$. Hence find the exact coordinates of the turning point P.

State the range of the function $\mathrm{f}(x)$, giving your answer exactly. [8]

(iii) Using the substitution $u = 2 - \cos x$ or otherwise, find the exact value of $\displaystyle\int_0^{\pi} \frac{\sin x}{2 - \cos x}\,\mathrm{d}x$. [4]

(iv) Sketch the graph of $y = \mathrm{f}(2x)$. [1]

(v) Using your answers to parts **(iii)** and **(iv)**, write down the exact value of $\displaystyle\int_0^{\frac{1}{2}\pi} \frac{\sin 2x}{2 - \cos 2x}\,\mathrm{d}x$. [2]