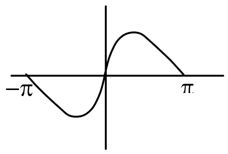
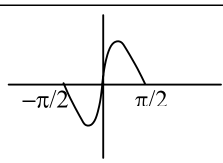
Section B

7(i) $2x - x\ln x = 0$ $\Rightarrow$ $x(2 - \ln x) = 0$ $\Rightarrow$ $(x = 0)$ or $\ln x = 2$ $\Rightarrow$ at A, $x = e^2$	M1 A1 [2]	Equating to zero
(ii) $\frac{dy}{dx} = 2 - x.\frac{1}{x} - \ln x.1$ $= 1 - \ln x$ $\frac{dy}{dx} = 0 \Rightarrow 1 - \ln x = 0$ $\Rightarrow$ $\ln x = 1, x = e$ When $x = e$, $y = 2e - e\ln e = e$ So B is (e, e)	M1 B1 A1 M1 A1cao B1ft [6]	Product rule for $x \ln x$ $d/dx(\ln x) = 1/x$ $1 - \ln x$ o.e. equating their derivative to zero $x = e$ $y = e$
(iii) At A, $\frac{dy}{dx} = 1 - \ln e^2 = 1 - 2$ $= -1$ At C, $\frac{dy}{dx} = 1 - \ln 1 = 1$ $1 \times -1 = -1 \Rightarrow$ tangents are perpendicular	M1 A1cao E1 [3]	Substituting x=1 or their e^2 into their derivative -1 and 1 www
(iv) Let $u = \ln x$, $dv/dx = x$ $\Rightarrow v = ½x^2$ $\int x\ln x\,dx = \frac{1}{2}x^2\ln x - \int \frac{1}{2}x^2.\frac{1}{x}dx$ $= \frac{1}{2}x^2\ln x - \frac{1}{2}\int x\,dx$ $= \frac{1}{2}x^2\ln x - \frac{1}{4}x^2 + c$ * $A = \int_1^e (2x - x\ln x)dx$ $= \left[x^2 - \frac{1}{2}x^2\ln x + \frac{1}{4}x^2\right]_1^e$ $= (e^2 - ½e^2\ln e + ¼e^2) - (1 - ½1^2\ln 1 + ¼1^2)$ $= \frac{3}{4}e^2 - \frac{5}{4}$	M1 A1 E1 B1 B1 M1 A1 cao [7]	Parts: $u = \ln x$, $dv/dx = x \Rightarrow v = ½x^2$ correct integral and limits $\left[x^2 - \frac{1}{2}x^2\ln x + \frac{1}{4}x^2\right]$ o.e. substituting limits correctly

8 (i) $f(-x) = \dfrac{\sin(-x)}{2-\cos(-x)}$ $= \dfrac{-\sin(x)}{2-\cos(x)}$ $= -f(x)$ 	M1 A1 B1 [3]	substituting $-x$ for x in $f(x)$ Graph completed with rotational symmetry about O.
(ii) $f'(x) = \dfrac{(2-\cos x)\cos x - \sin x.\sin x}{(2-\cos x)^2}$ $= \dfrac{2\cos x - \cos^2 x - \sin^2 x}{(2-\cos x)^2}$ $= \dfrac{2\cos x - 1}{(2-\cos x)^2}$ * $f'(x) = 0$ when $2\cos x - 1 = 0$ $\Rightarrow \cos x = ½, x = \pi/3$ When $x = \pi/3$, $y = \dfrac{\sin(\pi/3)}{2-\cos(\pi/3)} = \dfrac{\sqrt{3}/2}{2-1/2}$ $= \dfrac{\sqrt{3}}{3}$ So range is $-\dfrac{\sqrt{3}}{3} \le y \le \dfrac{\sqrt{3}}{3}$	M1 A1 E1 M1 A1 M1 A1 B1ft [8]	Quotient or product rule consistent with their derivatives Correct expression numerator = 0 Substituting their $\pi/3$ into y o.e. but exact ft their $\dfrac{\sqrt{3}}{3}$
(iii) $\int_0^{\pi} \dfrac{\sin x}{2-\cos x}dx$ let $u = 2 - \cos x$ $\Rightarrow du/dx = \sin x$ When $x = 0$, $u = 1$; when $x = \pi$, $u = 3$ $= \int_1^3 \dfrac{1}{u}du$ $= [\ln u]_1^3$ $= \ln 3 - \ln 1 = \ln 3$	M1 B1 A1ft A1cao	$\int \dfrac{1}{u}du$ $u = 1$ to 3 $[\ln u]$
or $= [\ln(2-\cos x)]_0^{\pi}$ $= \ln 3 - \ln 1 = \ln 3$	M2 A1 A1 cao [4]	$[k \ln (2 - \cos x)]$ $k = 1$
(iv) 	B1ft [1]	Graph showing evidence of stretch s.f. ½ in x – direction
(v) Area is stretched with scale factor ½ So area is ½ ln 3	M1 A1ft [2]	soi ½ their ln 3