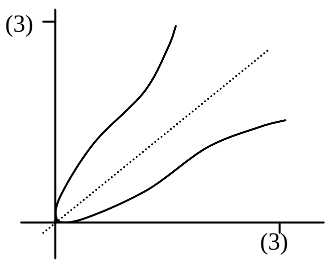
Section B

8 (i) At P, $x \sin 3x = 0$ $\Rightarrow \sin 3x = 0$ $\Rightarrow 3x = \pi$ $\Rightarrow x = \pi/3$	M1 A1 A1cao [3]	$x \sin 3x = 0$ $3x = \pi$ or 180 $x = \pi/3$ or 1.05 or better
(ii) When $x = \pi/6$, $x \sin 3x = \frac{\pi}{6}\sin\frac{\pi}{2} = \frac{\pi}{6}$ $\Rightarrow$ Q($\pi/6$, $\pi/6$) lies on line $y = x$	E1 [1]	$y = \frac{\pi}{6}$ or $x \sin 3x = x \Rightarrow \sin 3x = 1$ etc. Must conclude in radians, and be exact
(iii) $y = x\sin 3x$ $\Rightarrow \frac{dy}{dx} = x.3\cos 3x + \sin 3x$ At Q, $\frac{dy}{dx} = \frac{\pi}{6}.3\cos\frac{\pi}{2} + \sin\frac{\pi}{2} = 1$ = gradient of $y = x$ So line touches curve at this point	B1 M1 A1cao M1 A1ft E1 [6]	d/dx (sin 3x) = 3cos 3x Product rule consistent with their derivs $3x \cos 3x + \sin 3x$ substituting $x = \pi/6$ into their derivative = 1 ft dep 1st M1 = gradient of $y = x$ (www)
(iv) Area under curve = $\int_0^{\frac{\pi}{6}} x\sin 3x dx$ Integrating by parts, $u = x$, dv/dx = sin3x $\Rightarrow v = -\frac{1}{3}\cos 3x$ $\int_0^{\frac{\pi}{6}} x\sin 3x dx = \left[-\frac{1}{3}x\cos 3x\right]_0^{\frac{\pi}{6}} + \int_0^{\frac{\pi}{6}} \frac{1}{3}\cos 3x dx$ $= -\frac{1}{3}.\frac{\pi}{6}\cos\frac{\pi}{2} + \frac{1}{3}.0.\cos 0 + \left[\frac{1}{9}\sin 3x\right]_0^{\frac{\pi}{6}}$ $= \frac{1}{9}$ Area under line = $\frac{1}{2}\times\frac{\pi}{6}\times\frac{\pi}{6} = \frac{\pi^2}{72}$ So area required = $\frac{\pi^2}{72} - \frac{1}{9}$ $= \frac{\pi^2 - 8}{72}$ *	 M1 A1cao A1ft M1 A1 B1 E1 [7]	 Parts with $u = x$ dv/dx = sin 3x $\Rightarrow$ $v = -\frac{1}{3}\cos 3x$ [condone no negative] $\ldots + \left[\frac{1}{9}\sin 3x\right]_0^{\frac{\pi}{6}}$ substituting (correct) limits $\frac{1}{9}$ www $\frac{\pi^2}{72}$ www

9 (i) $f(-x) = \ln[1 + (-x)^2]$ $= \ln[1 + x^2] = f(x)$ Symmetrical about Oy	M1 E1 B1 [3]	If verifies that $f(-x) = f(x)$ using a particular point, allow SCB1 For $f(-x) = \ln(1 + x^2) = f(x)$ allow M1E0 For $f(-x) = \ln(1 +- x^2) = f(x)$ allow M1E0 or 'reflects in Oy', etc
(ii) $y = \ln(1 + x^2)$ let $u = 1 + x^2$ $dy/du = 1/u$, $du/dx = 2x$ $\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$ $= \frac{1}{u}.2x = \frac{2x}{1+x^2}$ When $x = 2$, $dy/dx = 4/5$.	M1 B1 A1 A1cao [4]	Chain rule $1/u$ soi
(iii) The function is not one to one for this domain	B1 [1]	Or many to one
(iv)  Domain for g(x) $= 0 \le x \le \ln 10$ $y = \ln(1 + x^2)$ $x \leftrightarrow y$ $x = \ln(1 + y^2)$ $\Rightarrow$ $e^x = 1 + y^2$ $\Rightarrow$ $e^x - 1 = y^2$ $\Rightarrow$ $y = \sqrt{(e^x - 1)}$ so $g(x) = \sqrt{(e^x - 1)}$* *or* $g\,f(x) = g[\ln(1 + x^2)]$ $= \sqrt{e^{\ln(1+x^2)} - 1}$ $= (1 + x^2) - 1$ $= x$	M1 A1 B1 M1 M1 E1 M1 M1 E1 [6]	g(x) is f(x) reflected in $y = x$ Reasonable shape and domain, i.e. no –ve x values, inflection shown, does not cross $y = x$ line Condone y instead of x Attempt to invert function Taking exponentials $g(x) = \sqrt{(e^x - 1)}$* www forming g f(x) or f g(x) $e^{\ln(1+x^2)} = 1 + x^2$ or $\ln(1 + e^x - 1) = x$ www
(v) $g'(x) = ½ (e^x - 1)^{-1/2} . e^x$ $\Rightarrow g'(\ln 5) = ½ (e^{\ln 5} - 1)^{-1/2} . e^{\ln 5}$ $= ½ (5 - 1)^{-1/2} . 5$ $= 5/4$ Reciprocal of gradient at P as tangents are reflections in $y = x$.	B1 B1 M1 E1cao B1 [5]	$½ u^{-1/2}$ soi $\times e^x$ substituting ln 5 into g′ - must be some evidence of substitution Must have idea of reciprocal. Not 'inverse'.