

Binomial Expansions

January 2007
6666 Core Mathematics C4
Mark Scheme

Question Number	Scheme	Marks
1.	<p>** represents a constant</p> $f(x) = (2 - 5x)^{-2} = \underline{(2)^{-2}} \left(1 - \frac{5x}{2}\right)^{-2} = \underline{\frac{1}{4}} \left(1 - \frac{5x}{2}\right)^{-2}$ <p>Takes 2 outside the bracket to give any of $(2)^{-2}$ or $\frac{1}{4}$.</p> $= \underline{\frac{1}{4} \left\{ 1 + (-2)(**x) + \frac{(-2)(-3)}{2!} (**x)^2 + \frac{(-2)(-3)(-4)}{3!} (**x)^3 + \dots \right\}}$ <p>Expands $(1 + **x)^{-2}$ to give an unsimplified $1 + (-2)(**x)$;</p> <p>A correct unsimplified {.....} expansion with candidate's $(**x)$</p> $= \underline{\frac{1}{4} \left\{ 1 + (-2)\left(\frac{-5x}{2}\right) + \frac{(-2)(-3)}{2!} \left(\frac{-5x}{2}\right)^2 + \frac{(-2)(-3)(-4)}{3!} \left(\frac{-5x}{2}\right)^3 + \dots \right\}}$ $= \frac{1}{4} \left\{ 1 + 5x; + \frac{75x^2}{4} + \frac{125x^3}{2} + \dots \right\}$ <p>Anything that cancels to $\frac{1}{4} + \frac{5x}{4}$;</p> <p>Simplified $\frac{75x^2}{16} + \frac{125x^3}{8}$</p> $= \frac{1}{4} + 1\frac{1}{4}x; + 4\frac{11}{16}x^2 + 15\frac{5}{8}x^3 + \dots$	B1 M1 A1 A1; A1

[5]

5 marks

Question Number	Scheme	Marks
Aliter 1. Way 2	$f(x) = (2 - 5x)^{-2}$ $= \left\{ (2)^{-2} + (-2)(2)^{-3}(**x) + \frac{(-2)(-3)}{2!}(2)^{-4}(**x)^2 \right. \\ \left. + \frac{(-2)(-3)(-4)}{3!}(2)^{-5}(**x)^3 + \dots \right\}$ $= \left\{ (2)^{-2} + (-2)(2)^{-3}(-5x) + \frac{(-2)(-3)}{2!}(2)^{-4}(-5x)^2 \right. \\ \left. + \frac{(-2)(-3)(-4)}{3!}(2)^{-5}(-5x)^3 + \dots \right\}$ $= \left\{ \frac{1}{4} + (-2)(\frac{1}{8})(-5x) + (3)(\frac{1}{16})(25x^2) \right. \\ \left. + (-4)(\frac{1}{16})(-125x^3) + \dots \right\}$ $= \frac{1}{4} + \frac{5x}{4} + \frac{75x^2}{16} + \frac{125x^3}{8} + \dots$ $= \frac{1}{4} + 1\frac{1}{4}x + 4\frac{11}{16}x^2 + 15\frac{5}{8}x^3 + \dots$	$\frac{1}{4}$ or $(2)^{-2}$ B1 Expands $(2 - 5x)^{-2}$ to give an unsimplified $(2)^{-2} + (-2)(2)^{-3}(**x)$; M1 A correct unsimplified {.....} expansion with candidate's $(**x)$ A1 Anything that cancels to $\frac{1}{4} + \frac{5x}{4}$; A1; Simplified $\frac{75x^2}{16} + \frac{125x^3}{8}$ A1 [5] 5 marks

Attempts using Maclaurin expansions need to be referred to your team leader.

June 2007
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Question Number	Scheme	Marks
1. (a)	<p>** represents a constant</p> $f(x) = (3 + 2x)^{-3} = \underline{(3)^{-3}} \left(1 + \frac{2x}{3}\right)^{-3} = \frac{1}{27} \left(1 + \frac{2x}{3}\right)^{-3}$ <p>$= \frac{1}{27} \left\{ 1 + (-3)(**x) + \frac{(-3)(-4)}{2!} (**x)^2 + \frac{(-3)(-4)(-5)}{3!} (**x)^3 + \dots \right\}$</p> <p>with $** \neq 1$</p> $= \frac{1}{27} \left\{ 1 + (-3)\left(\frac{2x}{3}\right) + \frac{(-3)(-4)}{2!} \left(\frac{2x}{3}\right)^2 + \frac{(-3)(-4)(-5)}{3!} \left(\frac{2x}{3}\right)^3 + \dots \right\}$ $= \frac{1}{27} \left\{ 1 - 2x + \frac{8x^2}{3} - \frac{80}{27} x^3 + \dots \right\}$ $= \frac{1}{27} - \frac{2x}{27} + \frac{8x^2}{81} - \frac{80x^3}{729} + \dots$ <p>Anything that cancels to $\frac{1}{27} - \frac{2x}{27}$, Simplified $\frac{8x^2}{81} - \frac{80x^3}{729}$</p>	B1 M1; A1 √ A1; A1 [5] 5 marks

Note: You would award: B1M1A0 for

$$= \frac{1}{27} \left\{ 1 + (-3)\left(\frac{2x}{3}\right) + \frac{(-3)(-4)}{2!} (2x)^2 + \frac{(-3)(-4)(-5)}{3!} (2x)^3 + \dots \right\}$$

because ** is not consistent.

Special Case: If you see the constant $\frac{1}{27}$ in a candidate's final binomial expression, then you can award B1

Question Number	Scheme	Marks
<p>Aliter 1. Way 2</p> $f(x) = (3 + 2x)^{-3}$ $= \left\{ (3)^{-3} + (-3)(3)^{-4}(**x) + \frac{(-3)(-4)}{2!}(3)^{-5}(**x)^2 \right.$ $\quad \left. + \frac{(-3)(-4)(-5)}{3!}(3)^{-6}(**x)^3 + \dots \right\}$ <p>with $** \neq 1$</p> $= \left\{ (3)^{-3} + (-3)(3)^{-4}(2x) + \frac{(-3)(-4)}{2!}(3)^{-5}(2x)^2 \right.$ $\quad \left. + \frac{(-3)(-4)(-5)}{3!}(3)^{-6}(2x)^3 + \dots \right\}$ $= \left\{ \frac{1}{27} + (-3)\left(\frac{1}{81}\right)(2x) + (6)\left(\frac{1}{243}\right)(4x^2) \right.$ $\quad \left. + (-10)\left(\frac{1}{729}\right)(8x^3) + \dots \right\}$ $= \frac{1}{27} - \frac{2x}{27} + \frac{8x^2}{81} - \frac{80x^3}{729} + \dots$	<p>$\frac{1}{27}$ or $(3)^{-3}$ (See note ↓) Expands $(3 + 2x)^{-3}$ to give an un-simplified or simplified $(3)^{-3} + (-3)(3)^{-4}(**x)$; A correct un-simplified or simplified $\{\dots\}$ expansion with candidate's followed thro' $(**x)$</p> <p>Anything that cancels to $\frac{1}{27} - \frac{2x}{27}$, Simplified $\frac{8x^2}{81} - \frac{80x^3}{729}$</p>	B1 M1 A1 √ A1; A1 [5] 5 marks

Attempts using Maclaurin expansions need to be escalated up to your team leader.

If you feel the mark scheme does not apply fairly to a candidate please escalate the response up to your team leader.

Special Case: If you see the constant $\frac{1}{27}$ in a candidate's final binomial expression, then you can award B1

Question Number	Scheme	Marks
2. (a)	<p>** represents a constant (which must be consistent for first accuracy mark)</p> $(8-3x)^{\frac{1}{3}} = \underline{(8)^{\frac{1}{3}}} \left(1 - \frac{3x}{8}\right)^{\frac{1}{3}} = \underline{2} \left(1 - \frac{3x}{8}\right)^{\frac{1}{3}}$ $= 2 \left\{ 1 + (\frac{1}{3})(**x) + \frac{(\frac{1}{3})(-\frac{2}{3})}{2!} (**x)^2 + \frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!} (**x)^3 + \dots \right\}$ <p style="text-align: center;">with $** \neq 1$</p> $= 2 \left\{ 1 + (\frac{1}{3})(-\frac{3x}{8}) + \frac{(\frac{1}{3})(-\frac{2}{3})}{2!} (-\frac{3x}{8})^2 + \frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!} (-\frac{3x}{8})^3 + \dots \right\}$ $= 2 \left\{ 1 - \frac{1}{8}x; -\frac{1}{64}x^2 - \frac{5}{1536}x^3 - \dots \right\}$ $= 2 - \frac{1}{4}x; -\frac{1}{32}x^2 - \frac{5}{768}x^3 - \dots$	<p>Takes 8 outside the bracket to give any of $\underline{(8)^{\frac{1}{3}}}$ or $\underline{2}$.</p> <p>Expands $(1 + **x)^{\frac{1}{3}}$ to give a simplified or an un-simplified $1 + (\frac{1}{3})(**x)$;</p> <p>A correct simplified or an un-simplified $\{.....\}$ expansion with candidate's followed through $(**x)$</p> <p>Award SC M1 if you see $\frac{(\frac{1}{3})(-\frac{2}{3})}{2!} (**x)^2 + \frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!} (**x)^3$</p> <p>Either $2\{1 - \frac{1}{8}x \dots\}$ or anything that cancels to $2 - \frac{1}{4}x$; Simplified $-\frac{1}{32}x^2 - \frac{5}{768}x^3$</p>
(b)	$(7.7)^{\frac{1}{3}} \approx 2 - \frac{1}{4}(0.1) - \frac{1}{32}(0.1)^2 - \frac{5}{768}(0.1)^3 - \dots$ $= 2 - 0.025 - 0.0003125 - 0.0000065104166\dots$ $= 1.97468099\dots$	<p>Attempt to substitute $x=0.1$ into a candidate's binomial expansion.</p> <p>awrt 1.9746810</p>

You would award B1M1A0 for

$$= 2 \left\{ 1 + (\frac{1}{3})(-\frac{3x}{8}) + \frac{(\frac{1}{3})(-\frac{2}{3})}{2!} (-\frac{3x}{8})^2 + \frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!} (-3x)^3 + \dots \right\}$$

because ** is not consistent.

If you see the constant term "2" in a candidate's final binomial expansion, then you can award B1.

Be wary of calculator value of $(7.7)^{\frac{1}{3}} = 1.974680822\dots$

Question Number	Scheme	Marks
<p>Aliter 2. (a) Way 2</p> <p>$(8 - 3x)^{\frac{1}{3}}$</p> $= \left\{ (8)^{\frac{1}{3}} + \left(\frac{1}{3}\right)(8)^{-\frac{2}{3}}(* * x) ; + \frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)}{2!}(8)^{-\frac{5}{3}}(* * x)^2 \right.$ $\left. + \frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)}{3!}(8)^{-\frac{8}{3}}(* * x)^3 + \dots \right\}$ <p>with $* * \neq 1$</p> $= \left\{ (8)^{\frac{1}{3}} + \left(\frac{1}{3}\right)(8)^{-\frac{2}{3}}(-3x) ; + \frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)}{2!}(8)^{-\frac{5}{3}}(-3x)^2 \right.$ $\left. + \frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)}{3!}(8)^{-\frac{8}{3}}(-3x)^3 + \dots \right\}$ $= \left\{ 2 + \left(\frac{1}{3}\right)\left(\frac{1}{4}\right)(-3x) + \left(-\frac{1}{9}\right)\left(\frac{1}{32}\right)(9x^2) + \left(\frac{5}{81}\right)\left(\frac{1}{256}\right)(-27x^3) + \dots \right\}$ $= 2 - \frac{1}{4}x; - \frac{1}{32}x^2 - \frac{5}{768}x^3 - \dots$	<p>2 or $(8)^{\frac{1}{3}}$ (See note ↓)</p> <p>Expands $(8 - 3x)^{\frac{1}{3}}$ to give an un-simplified or simplified $(8)^{\frac{1}{3}} + \left(\frac{1}{3}\right)(8)^{-\frac{2}{3}}(* * x)$;</p> <p>A correct un-simplified or simplified $\{.....\}$ expansion with candidate's followed through $(* * x)$</p> <p>Award SC M1 if you see</p> $\frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)}{2!}(8)^{-\frac{5}{3}}(* * x)^2$ $+ \frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)}{3!}(8)^{-\frac{8}{3}}(* * x)^3$ <p>Anything that cancels to $2 - \frac{1}{4}x$; or $2\left\{1 - \frac{1}{8}x \dots \dots \right\}$</p> <p>Simplified $- \frac{1}{32}x^2 - \frac{5}{768}x^3$</p>	B1 M1; A1 √ A1; A1 [5]

Attempts using Maclaurin expansion should be escalated up to your team leader.

Be wary of calculator value of $(7.7)^{\frac{1}{3}} = 1.974680822\dots$

If you see the constant term “2” in a candidate’s final binomial expansion, then you can award B1.

Question Number	Scheme	Marks
5. (a)	<p>** represents a constant (which must be consistent for first accuracy mark)</p> $\frac{1}{\sqrt{(4-3x)}} = (4-3x)^{-\frac{1}{2}} = \underline{(4)^{-\frac{1}{2}}} \left(1 - \frac{3x}{4}\right)^{-\frac{1}{2}} = \frac{1}{2} \left(1 - \frac{3x}{4}\right)^{-\frac{1}{2}}$ <p style="text-align: right;"><u>(4)^{-1/2}</u> or <u>1/2</u> outside brackets</p> $= \frac{1}{2} \left[1 + \left(-\frac{1}{2}\right)(**x) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2!} (**x)^2 + \dots \right]$ <p style="text-align: center;">with $** \neq 1$</p> $= \frac{1}{2} \left[1 + \left(-\frac{1}{2}\right)\left(-\frac{3x}{4}\right) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2!} \left(-\frac{3x}{4}\right)^2 + \dots \right]$ $= \frac{1}{2} \left[1 + \frac{3}{8}x + \frac{27}{128}x^2 + \dots \right]$ $\left\{ = \frac{1}{2} + \frac{3}{16}x + \frac{27}{256}x^2 + \dots \right\}$	<p><u>B1</u></p> <p>Expands $(1+**x)^{-\frac{1}{2}}$ to give a simplified or an un-simplified $1 + (-\frac{1}{2})(**x)$;</p> <p>A correct simplified or an un-simplified [.....] expansion with candidate's followed through $(**x)$</p> <p>Award SC M1 if you see $(-\frac{1}{2})(**x) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2!} (**x)^2$</p> <p>$\frac{1}{2} \left[1 + \frac{3}{8}x; \dots \right]$</p> <p>SC: $K \left[1 + \frac{3}{8}x + \frac{27}{128}x^2 + \dots \right]$</p> <p>$\frac{1}{2} \left[\dots; \frac{27}{128}x^2 \right]$</p> <p><i>Ignore subsequent working</i></p>
(b)	$(x+8) \left(\frac{1}{2} + \frac{3}{16}x + \frac{27}{256}x^2 + \dots \right)$ $= \frac{1}{2}x + \frac{3}{16}x^2 + \dots$ $+ 4 + \frac{3}{2}x + \frac{27}{32}x^2 + \dots$ $= 4 + 2x; + \frac{33}{32}x^2 + \dots$	<p>Writing $(x+8)$ multiplied by candidate's part (a) expansion.</p> <p>Multiply out brackets to find a constant term, two x terms and two x^2 terms.</p> <p>Anything that cancels to $4 + 2x; \frac{33}{32}x^2$</p> <p>M1</p> <p>M1</p> <p>A1; A1</p> <p>[4]</p> <p>9 marks</p>

June 2009
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Question Number	Scheme	Marks
Q1	$f(x) = \frac{1}{\sqrt{4+x}} = (4+x)^{-\frac{1}{2}}$ $= (4)^{-\frac{1}{2}}(1 + \dots)^{-\frac{1}{2}} \quad \frac{1}{2}(1 + \dots)^{-\frac{3}{2}} \text{ or } \frac{1}{2\sqrt{1+\dots}}$ $= \dots \left(1 + \left(-\frac{1}{2} \right) \left(\frac{x}{4} \right) + \frac{\left(-\frac{1}{2} \right) \left(-\frac{3}{2} \right)}{2} \left(\frac{x}{4} \right)^2 + \frac{\left(-\frac{1}{2} \right) \left(-\frac{3}{2} \right) \left(-\frac{5}{2} \right)}{3!} \left(\frac{x}{4} \right)^3 + \dots \right)$ $\text{ft their } \left(\frac{x}{4} \right)$ $= \frac{1}{2} - \frac{1}{16}x + \frac{3}{256}x^2 - \frac{5}{2048}x^3 + \dots$	M1 B1 M1 A1ft A1, A1 (6) [6]
	<i>Alternative</i> $f(x) = \frac{1}{\sqrt{4+x}} = (4+x)^{-\frac{1}{2}}$ $= 4^{-\frac{1}{2}} + \left(-\frac{1}{2} \right) 4^{-\frac{3}{2}} x + \frac{\left(-\frac{1}{2} \right) \left(-\frac{3}{2} \right)}{1.2} 4^{-\frac{5}{2}} x^2 + \frac{\left(-\frac{1}{2} \right) \left(-\frac{3}{2} \right) \left(-\frac{5}{2} \right)}{1.2.3} 4^{-\frac{7}{2}} x^3 + \dots$ $= \frac{1}{2} - \frac{1}{16}x + \frac{3}{256}x^2 - \frac{5}{2048}x^3 + \dots$	M1 <u>B1</u> M1 A1 A1, A1 (6)

**January 2010
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Q1	<p>(a) $(1-8x)^{\frac{1}{2}} = 1 + \left(\frac{1}{2}\right)(-8x) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2}(-8x)^2 + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}(-8x)^3 + \dots$ $= 1 - 4x - 8x^2; -32x^3 - \dots$</p> <p>(b) $\sqrt{(1-8x)} = \sqrt{\left(1 - \frac{8}{100}\right)}$ $= \sqrt{\frac{92}{100}} = \sqrt{\frac{23}{25}} = \frac{\sqrt{23}}{5} *$</p> <p>(c) $1 - 4x - 8x^2 - 32x^3 = 1 - 4(0.01) - 8(0.01)^2 - 32(0.01)^3$ $= 1 - 0.04 - 0.0008 - 0.000032 = 0.959168$</p> <p>$\sqrt{23} = 5 \times 0.959168$ $= 4.79584$</p>	M1 A1 A1; A1 (4) M1 cso A1 (2) M1 cao A1 (3) [9]