Arithmetic Progressions APs

Consider $S_7 = 1 + 2 + 3 + 4 + 5 + 6 + 7$

$S_7 = 7 + 6 + 5 + 4 + 3 + 2 + 1$

Add $2S_7 = 8 + 8 + 8 + 8 + 8 + 8 + 8$

$2S_7 = 7 \times 8$

$$S_7 = \frac{7 \times 8}{2} = 28$$

$$S_{100} = \frac{100 \times 101}{2} = \frac{10100}{2} = 5050$$

A.P. a = 1st term d = common difference

Term

1st $= a$

2nd $= a + d$

3rd $= a + 2d$

4th $= a + 3d$

5th $= a + 4d$

$n^{th} = a + (n-1)d$

S_n = the sum of first n terms

$S_n = a + a+d + a+2d + \dots + a+(n-2)d + a+(n-1)d$

$S_n = a+(n-1)d + a+(n-2)d + \dots + a+d + a$

$2S_n = 2a+(n-1)d + 2a+(n-1)d + \dots + 2a+(n-1)d + 2a+(n-1)d$

$$2S_n = n(2a + (n-1)d)$$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

Sometimes we consider the nth term as the last term L

$$L = a + (n-1)d$$

$$S_n = \frac{n}{2}(a + L)$$

Example 1

The 4th term of an AP is 16 and the 7th term is 25. Find the first term a, the common difference d, and the sum of the first 10 terms S_{10}

$$4^{th} = a + 3d = 16 \quad (1)$$
$$7^{th} = a + 6d = 25 \quad (2)$$

(2) - (1)

$$3d = 9$$
$$\underline{d = 3}$$

Sub in (1)

$$a + 3(3) = 16$$
$$a + 9 = 16$$
$$a = 16 - 9$$
$$a = 7$$

Sequence would be 7, 10, 13, 16, ...

$$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$$

$$S_{10} = \frac{10}{2}\left(2(7) + 9(3)\right)$$

$$= 5(41)$$

$$= 205$$

Example 2

Which term of the sequence 15, 18, 21, 24, 27, 30, ... is equal to 4152 ?

$$n^{th} \text{ term} = a + (n-1)d$$

First find a and d

$$a = 15, \quad d = 3$$

$$a + (n-1)d = 4152$$

$$15 + (n-1) \times 3 = 4152$$

$$3(n-1) = 4137$$

$$n - 1 = \frac{4137}{3} = 1379$$

$$n = 1380$$

Example 3

Which term of the sequence 152, 145, 138, 124, ... is the first to have a negative value ?

$a = 152 \qquad d = -7$

n^{th} term $= a + (n-1)d$

Require $\quad a + (n-1)d < 0$

$$152 + (n-1)(-7) < 0$$

$$152 - 7n + 7 < 0$$

$$159 < 7n$$

$$\frac{159}{7} < n$$

$$n > 22\tfrac{5}{7}$$

$$n = 23$$

23rd term is first negative term

Example 4

Granny gives John £10 on his 1st birthday and on each birthday she gives him £5 more than the previous birthday. On which birthday does the total amount John has received since birth exceed £250?

$a = 10 \qquad d = 5$

$$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$$

$$\frac{n}{2}\left(2a + (n-1)d\right) > 250$$

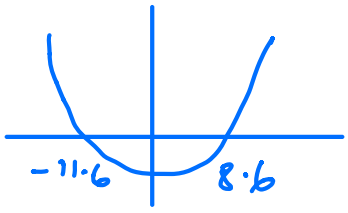
$$n\left(20 + 5(n-1)\right) > 500$$

$$n\left(20 + 5n - 5\right) > 500$$

$$n(15+5n) > 500$$

$$5n^2 + 15n - 500 > 0$$

$$n^2 + 3n - 100 > 0$$



$$(n - 8.611)(n + 11.611) > 0$$

$$n > 8.6$$

$$\underline{n = 9}$$