

# MEI Exercise 4D

4)  $y = xe^x$

i)  $\frac{dy}{dx} = xe^x + e^x = e^x(x+1)$

$$\frac{d^2y}{dx^2} \quad e^x + (x+1)e^x = (x+2)e^x$$


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ii) At min pt  $\frac{dy}{dx} = 0$

$$\Rightarrow e^x(x+1) = 0$$

$$\begin{aligned}\Rightarrow x+1 &= 0 \\ x &= -1\end{aligned}$$

$$y = -1e^{-1} = -e^{-1} = -\frac{1}{e}$$

$$\text{Min pt } \left(-1, -\frac{1}{e}\right)$$


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a)  $P = Ae^{kt}$

$$\frac{dP}{dt} = Ake^{kt}$$

$$\left. \begin{cases} t = 0 \\ P = 3 \\ \frac{dP}{dt} = 6 \end{cases} \right\}$$

$$] = Ae^0 \Rightarrow A = 3$$

$$t=0 \quad \text{Solve in } \frac{dp}{dt} \quad p = 3ke^0 \\ \Rightarrow k=2$$

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b)  $p = 3e^{2t}$

Find  $t$   
for  $p=12$

$$12 = 3e^{2t}$$
$$4 = e^{2t}$$
$$\ln 4 = \ln(e^{2t})$$

$$\ln 4 = 2t \ln e$$

$$\ln 4 = 2t$$

$$t = \frac{1}{2} \ln 4$$

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$$t = 0.693 \text{ hrs}$$

c)  $p \rightarrow \infty$  as  $t \rightarrow \infty$   
growth is unbounded

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ii)  $f(x) = 2x - x \ln x$

i)  $f'(e^3) = 2e^3 - e^3 \ln e^3$   
 $= 2e^3 - e^3 \times 3$   
 $= -e^3$

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$$\text{i)} \quad f'(x) = 2 - \left( x \frac{1}{x} + \ln x \right)$$

$$= 2 - 1 - \ln x$$

$$= 1 - \ln x$$

$$\underline{f''(x) = -\frac{1}{x}}$$

$$\text{iii) At st.pt. } f'(x) = 0$$

$$\Rightarrow 1 - \ln x = 0$$

$$1 = \ln x$$

$$\Rightarrow x = e \quad \text{only solution}$$

so 1 st pt.

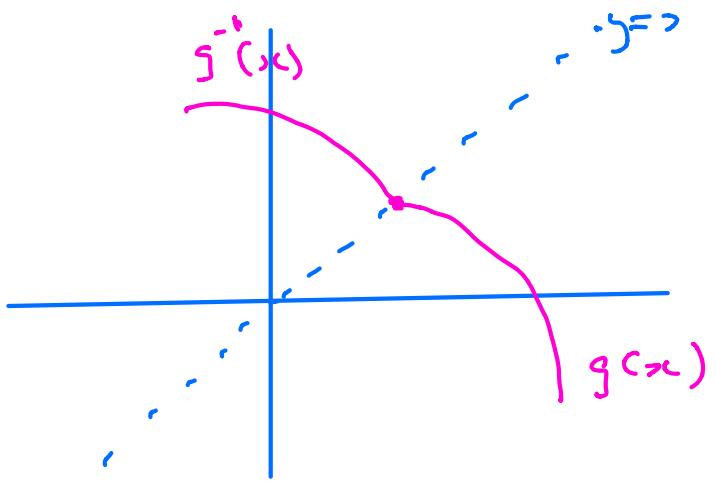
$$f(e) 2e - e \ln e = e$$

$$\underline{\text{st pt } (e, e)}$$

$$\underline{f''(e) = -\frac{1}{e} < 0}$$

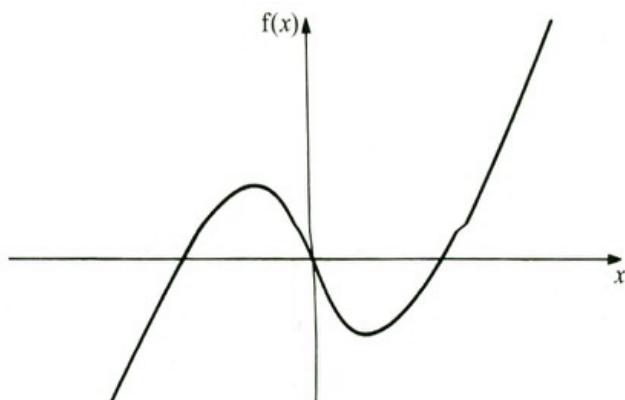
$$\text{iv) Range } g(x) \leq e$$

v)  $g(x)$  has an inverse since it is 1 to 1



## Exercise 5, 7, 8

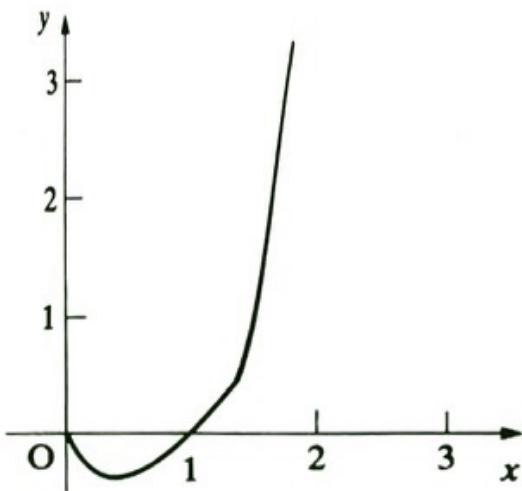
- 5 The graph of  $f(x) = x \ln(x^2)$  is shown below.



- (i) Describe, giving a reason, any symmetries of the graph.
- (ii) Find  $f'(x)$  and  $f''(x)$ .
- (iii) Find the co-ordinates of any stationary points.

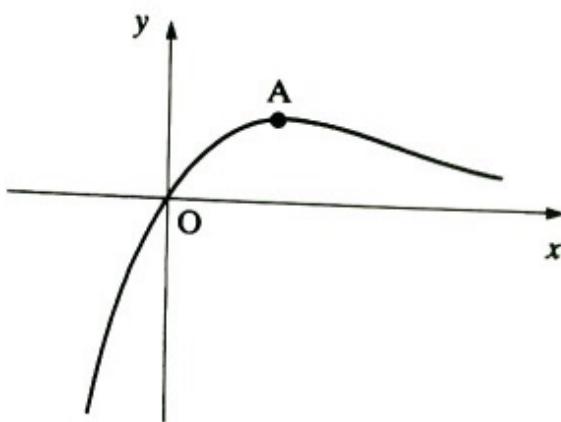
- 7 (i)** Differentiate  $\ln x$  and  $x \ln x$  with respect to  $x$ .

The sketch shows the graph of  $y = x \ln x$  for  $0 \leq x \leq 3$ .



- (ii)** Show that the curve has a stationary point  $\left(\frac{1}{e}, -\frac{1}{e}\right)$ .

- 8** The diagram shows the graph of  $y = xe^{-x}$ .



- (i)** Differentiate  $xe^{-x}$ .

- (ii)** Find the co-ordinates of the point A, the maximum point on the curve.