

Probability Generating Functions

If X has probability mass function $P(X=x)$,
then the probability generating function of X is
given by

$$G_X(t) = \sum P(X=x)t^x$$

where t is a dummy variable

For any probability generating function $G_X(t)=1$

Exercise 7A

1) $G_X(t) = 0.3 + 0.2t + 0.5t^2$

a) 0 1 2

b) $P(X=0) = 0.3, P(X > 0) = 1$

3) $G_Y(t) = 0.7 + 0.1(t^2 + t^3 + t^5)$

a) $P(Y=1) = 0$

b) $P(Y < 3) = 0.8$

c) $P(3 \leq Y \leq 6) = 0.2$

5) $G_X(t) = 0.4t + 0.2t^2 + 0.2t^3 + 0.2t^4$

Exercise 7B

1) a) $X \sim B(n, p)$ $G_X(t) = (1-p + pt)^n$

$$\Rightarrow X \sim (4, 0.5) = G_X(t) = (0.5 + 0.5t)^4$$

b) $X \sim P_0(\lambda)$ $G_X(t) = e^{\lambda(t-1)}$

$$X \sim P_0(1.7) \quad G_X(t) = e^{1.7(t-1)}$$

2) $X \sim Geo(p)$ $G_X(t) = \frac{pt}{1-(1-p)t}$

c) $X \sim Geo(0.3)$ $G_X(t) = \frac{0.3t}{1-0.7t}$

d) $X \sim \text{Negative}(r, p)$ $G_X(t) = \left(\frac{pt}{1-(1-p)t} \right)^r$

$$X \sim \text{Negative}(3, 0.4) \quad G_X(t) = \left(\frac{0.4t}{1-0.6t} \right)^3$$
