Appendix 2: Notation

The tables below set out the notation that must be used in A Level Mathematics examinations. Students will be expected to understand this notation without need for further explanation.

1		Set notation
1.1	$\in$	is an element of
1.2	$\notin$	is not an element of
1.3	$\subseteq$	is a subset of
1.4	$\subset$	is a proper subset of
1.5	$\{x_1, x_2, \ldots\}$	the set with elements $x_1, x_2, \ldots$
1.6	$\{x : \ldots\}$	the set of all x such that …
1.7	$n(A)$	the number of elements in set A
1.8	$\varnothing$	the empty set
1.9	ε	the universal set
1.10	A'	the complement of the set A
1.11	$\mathbb{N}$	the set of natural numbers, $\{1, 2, 3, \ldots\}$
1.12	$\mathbb{Z}$	the set of integers, $\{0, \pm 1, \pm 2, \pm 3, \ldots\}$
1.13	$\mathbb{Z}^+$	the set of positive integers, $\{1, 2, 3, \ldots\}$
1.14	$\mathbb{Z}_0^+$	the set of non-negative integers, $\{0, 1, 2, 3, \ldots\}$
1.15	$\mathbb{R}$	the set of real numbers
1.16	$\mathbb{Q}$	the set of rational numbers, $\left\{ \dfrac{p}{q} : p \in \mathbb{Z}, \, q \in \mathbb{Z}^+ \right\}$
1.17	$\cup$	union
1.18	$\cap$	intersection
1.19	(x, y)	the ordered pair x, y
1.20	$[a, b]$	the closed interval $\{x \in \mathbb{R}: a \le x \le b\}$
1.21	$[a, b)$	the interval $\{x \in \mathbb{R}: a \le x < b\}$
1.22	$(a, b]$	the interval $\{\{x \in \mathbb{R}: a < x \le b\}$
1.23	(a, b)	the open interval $\{x \in \mathbb{R}: a < x < b\}$

2		Miscellaneous symbols
2.1	$=$	is equal to
2.2	$\neq$	is not equal to
2.3	$\equiv$	is identical to or is congruent to
2.4	$\approx$	is approximately equal to
2.5	∞	infinity
2.6	$\propto$	is proportional to
2.7	$\therefore$	therefore
2.8	$\because$	because
2.9	$<$	is less than
2.10	$\leqslant$, $\leq$	is less than or equal to, is not greater than
2.11	$>$	is greater than
2.12	$\geqslant$, $\geq$	is greater than or equal to, is not less than
2.13	$p \Rightarrow q$	p implies q (if p then q)
2.14	$p \Leftarrow q$	p is implied by q (if q then p)
2.15	$p \Leftrightarrow q$	p implies and is implied by q (p is equivalent to q)
2.16	a	first term for an arithmetic or geometric sequence
2.17	l	last term for an arithmetic sequence
2.18	d	common difference for an arithmetic sequence
2.19	r	common ratio for a geometric sequence
2.20	S_n	sum to n terms of a sequence
2.21	S_∞	sum to infinity of a sequence

3		Operations
3.1	$a + b$	a plus b
3.2	$a - b$	a minus b
3.3	$a \times b$, ab, $a \cdot b$	a multiplied by b
3.4	$a \div b$, $\dfrac{a}{b}$	a divided by b
3.5	$\displaystyle\sum_{i=1}^{n} a_i$	$a_1 + a_2 + \ldots + a_n$
3.6	$\displaystyle\prod_{i=1}^{n} a_i$	$a_1 \times a_2 \times \ldots \times a_n$
3.7	$\sqrt{a}$	the non-negative square root of a
3.8	$\left| a \right|$	the modulus of a
3.9	$n!$	n factorial: $n! = n \times (n-1) \times \ldots \times 2 \times 1$, $n \in \mathbb{N}$; $0! = 1$
3.10	$\dbinom{n}{r}$, $^{n}C_{r}$, $_{n}C_{r}$	the binomial coefficient $\dfrac{n!}{r!(n-r)!}$ for $n, r \in \mathbb{Z}_0^+$, $r \leqslant n$ or $\dfrac{n(n-1)\ldots(n-r+1)}{r!}$ for $n \in \mathbb{Q}$, $r \in \mathbb{Z}_0^+$

4		Functions
4.1	$\mathrm{f}(x)$	the value of the function f at x
4.2	$\mathrm{f} : x \mapsto y$	the function f maps the element x to the element y
4.3	f^{-1}	the inverse function of the function f
4.4	gf	the composite function of f and g which is defined by $\mathrm{gf}(x) = \mathrm{g}(\mathrm{f}(x))$
4.5	$\displaystyle\lim_{x \to a} \mathrm{f}(x)$	the limit of $\mathrm{f}(x)$ as x tends to a
4.6	Δx, δx	an increment of x
4.7	$\dfrac{\mathrm{d}y}{\mathrm{d}x}$	the derivative of y with respect to x
4.8	$\dfrac{\mathrm{d}^n y}{\mathrm{d}x^n}$	the nth derivative of y with respect to x
4.9	$\mathrm{f}'(x)$, $\mathrm{f}''(x)$, $\ldots$, $\mathrm{f}^{(n)}(x)$	the first, second, ..., nth derivatives of $\mathrm{f}(x)$ with respect to x

4		Functions
4.10	$\dot{x},\ \ddot{x},\ \dots$	the first, second, ... derivatives of x with respect to t
4.11	$\displaystyle\int y \ \mathrm{d}x$	the indefinite integral of y with respect to x
4.12	$\displaystyle\int_a^b y \ \mathrm{d}x$	the definite integral of y with respect to x between the limits $x = a$ and $x = b$

5		Exponential and Logarithmic Functions
5.1	e	base of natural logarithms
5.2	e^x, exp x	exponential function of x
5.3	$\log_a x$	logarithm to the base a of x
5.4	$\ln x$, $\log_e x$	natural logarithm of x

6		Trigonometric Functions
6.1	sin, cos, tan, cosec, sec, cot	the trigonometric functions
6.2	$\sin^{-1}, \cos^{-1}, \tan^{-1}$ arcsin, arccos, arctan	the inverse trigonometric functions
6.3	$^\circ$	degrees
6.4	rad	radians

7		Vectors
7.1	$\mathbf{a}$, $\underline{a}$, $\underset{\sim}{a}$	the vector $\mathbf{a}$, $\underline{a}$, $\underset{\sim}{a}$; these alternatives apply throughout section 9
7.2	$\overrightarrow{AB}$	the vector represented in magnitude and direction by the directed line segment $\mathbf{AB}$
7.3	$\hat{\mathbf{a}}$	a unit vector in the direction of $\mathbf{a}$
7.4	$\mathbf{i, j, k}$	unit vectors in the directions of the cartesian coordinate axes
7.5	$\lvert\mathbf{a}\rvert$, a	the magnitude of $\mathbf{a}$
7.6	$\left\lvert\overrightarrow{AB}\right\rvert$, AB	the magnitude of $\overrightarrow{AB}$

7		Vectors
7.7	$\begin{pmatrix} a \\ b \end{pmatrix}, \quad a\mathbf{i} + b\mathbf{j}$	column vector and corresponding unit vector notation
7.8	$\mathbf{r}$	position vector
7.9	$\mathbf{s}$	displacement vector
7.10	$\mathbf{v}$	velocity vector
7.11	$\mathbf{a}$	acceleration vector

8		Probability and Statistics
8.1	A, B, C, etc.	events
8.2	$A \cup B$	union of the events A and B
8.3	$A \cap B$	intersection of the events A and B
8.4	$\mathrm{P}(A)$	probability of the event A
8.5	A'	complement of the event A
8.6	$\mathrm{P}(A\,|\,B)$	probability of the event A conditional on the event B
8.7	$X,\ Y,\ R$, etc.	random variables
8.8	$x,\ y,\ r$, etc.	values of the random variables X, Y, R etc.
8.9	$x_1,\ x_2,\ \dots$	observations
8.10	$f_1,\ f_2,\ \dots$	frequencies with which the observations $x_1,\ x_2,\ \dots$ occur
8.11	$\mathrm{p}(x),\ \mathrm{P}(X = x)$	probability function of the discrete random variable X
8.12	$p_1,\ p_2,\ \dots$	probabilities of the values $x_1,\ x_2,\ \dots$ of the discrete random variable X
8.13	$\mathrm{E}(X)$	expectation of the random variable X
8.14	$\mathrm{Var}(X)$	variance of the random variable X
8.15	$\sim$	has the distribution
8.16	$\mathrm{B}(n,\ p)$	binomial distribution with parameters n and p, where n is the number of trials and p is the probability of success in a trial
8.17	q	$q = 1 - p$ for binomial distribution
8.18	$\mathrm{N}(\mu,\ \sigma^2)$	Normal distribution with mean μ and variance σ^2

8		Probability and Statistics
8.19	$Z \sim \mathrm{N}(0,1)$	standard Normal distribution
8.20	ϕ	probability density function of the standardised Normal variable with distribution $\mathrm{N}(0, 1)$
8.21	Φ	corresponding cumulative distribution function
8.22	μ	population mean
8.23	σ^2	population variance
8.24	σ	population standard deviation
8.25	$\overline{x}$	sample mean
8.26	s^2	sample variance
8.27	s	sample standard deviation
8.28	H_0	Null hypothesis
8.29	H_1	Alternative hypothesis
8.30	r	product moment correlation coefficient for a sample
8.31	ρ	product moment correlation coefficient for a population

9		Mechanics
9.1	kg	kilograms
9.2	m	metres
9.3	km	kilometres
9.4	m/s, m s^{-1}	metres per second (velocity)
9.5	m/s^2, m s^{-2}	metres per second per second (acceleration)
9.6	F	Force or resultant force
9.7	N	Newton
9.8	N m	Newton metre (moment of a force)
9.9	t	time
9.10	s	displacement
9.11	u	initial velocity
9.12	v	velocity or final velocity
9.13	a	acceleration
9.14	g	acceleration due to gravity
9.15	μ	coefficient of friction